

FORCE CONTROL

Manipulator interaction with environment

Compliance control

Impedance control

Force control

Constrained motion

Natural and artificial constraints

Hybrid force/motion control

MANIPULATOR INTERACTION WITH ENVIRONMENT

- Constrained motion
- Motion control (?)
 - ★ positioning accuracy
 - ★ environment uncertainty
- Unstable contact
- Force measurements
- Control strategies
 - ★ *indirect*
 - ★ *direct*

COMPLIANCE CONTROL

- Effects of contact force on *operational space* motion control schemes

$$B(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + F\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{u} - \mathbf{J}^T(\mathbf{q})\mathbf{h}_e$$

★ at equilibrium

$$\mathbf{J}_A^T(\mathbf{q})\mathbf{K}_P\tilde{\mathbf{x}} = \mathbf{J}^T(\mathbf{q})\mathbf{h}_e$$

⇓

$$\tilde{\mathbf{x}} = \mathbf{K}_P^{-1}\mathbf{T}_A^T(\mathbf{x})\mathbf{h}_e = \mathbf{K}_P^{-1}\mathbf{h}_A$$

- $\mathbf{K}_P^{-1} \equiv$ control *active compliance*
 - ★ linear with respect to \mathbf{f}
 - ★ torsional with respect to μ
- If $\mathbf{h}_e \in \mathcal{N}(\mathbf{J}^T) \implies \tilde{\mathbf{x}} = \mathbf{0} \quad \mathbf{h}_e \neq \mathbf{0}$

Passive compliance

- ★ elementary displacement between two rigid bodies R and S elastically coupled (with respect to either reference frame)

$$d\mathbf{x}_{r,s} = \begin{bmatrix} d\mathbf{o}_{r,s} \\ \boldsymbol{\omega}_{r,s} dt \end{bmatrix} = \mathbf{v}_{r,s} dt$$

↓

$$\mathbf{h}_s = \begin{bmatrix} \mathbf{f}_s \\ \boldsymbol{\mu}_s \end{bmatrix} = \begin{bmatrix} \mathbf{K}_f & \mathbf{K}_c \\ \mathbf{K}_c^T & \mathbf{K}_\mu \end{bmatrix} \begin{bmatrix} d\mathbf{o}_{r,s} \\ \boldsymbol{\omega}_{r,s} dt \end{bmatrix} = \mathbf{K} d\mathbf{x}_{r,s} \quad \mathbf{h}_r = -\mathbf{h}_s$$

- *Stiffness* matrix \mathbf{K} (symmetric and positive semi-definite)
 - ★ $\mathbf{K}_f \equiv$ translational stiffness
 - ★ $\mathbf{K}_\mu \equiv$ torsional stiffness
 - ★ $\mathbf{K}_c \equiv$ coupling stiffness
- *Compliance* matrix \mathbf{C}

$$d\mathbf{x}_{r,s} = \mathbf{C} \mathbf{h}_s$$

- ★ Remote Center of Compliance (RCC)

Active compliance

- ★ end-effector position and orientation with respect to desired frame

$$\mathbf{T}_e^d = (\mathbf{T}_d)^{-1} \mathbf{T}_e = \begin{bmatrix} \mathbf{R}_e^d & \mathbf{o}_{d,e}^d \\ \mathbf{0}^T & 1 \end{bmatrix}$$

$$\mathbf{R}_e^d = \mathbf{R}_d^T \mathbf{R}_e \quad \mathbf{o}_{d,e}^d = \mathbf{R}_d^T (\mathbf{o}_e - \mathbf{o}_d)$$

⇓

$$\tilde{\mathbf{x}} = - \begin{bmatrix} \mathbf{o}_{d,e}^d \\ \phi_{d,e} \end{bmatrix}$$

$$\dot{\tilde{\mathbf{x}}} = -\mathbf{J}_{A_d}(\mathbf{q}, \tilde{\mathbf{x}}) \dot{\mathbf{q}}$$

$$\mathbf{J}_{A_d}(\mathbf{q}, \tilde{\mathbf{x}}) = \mathbf{T}_A^{-1}(\phi_{d,e}) \begin{bmatrix} \mathbf{R}_d^T & \mathbf{O} \\ \mathbf{O} & \mathbf{R}_d^T \end{bmatrix} \mathbf{J}(\mathbf{q})$$

- PD control with gravity compensation

$$\mathbf{u} = \mathbf{g}(\mathbf{q}) + \mathbf{J}_{A_d}^T(\mathbf{q}, \tilde{\mathbf{x}}) \mathbf{K}_P \tilde{\mathbf{x}} - \mathbf{J}_{A_d}^T \mathbf{K}_D \mathbf{J}_{A_d}(\mathbf{q}, \tilde{\mathbf{x}}) \dot{\mathbf{q}}$$

- Stability analysis

$$V = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{B}(\mathbf{q}) \dot{\mathbf{q}} + \frac{1}{2} \tilde{\mathbf{x}}^T \mathbf{K}_P \tilde{\mathbf{x}} > 0 \quad \forall \dot{\mathbf{q}}, \tilde{\mathbf{x}} \neq \mathbf{0},$$

- ★ at equilibrium

$$\mathbf{J}_{A_d}^T(\mathbf{q}) \mathbf{K}_P \tilde{\mathbf{x}} = \mathbf{J}^T(\mathbf{q}) \mathbf{h}_e$$

$$\mathbf{h}_e^d = \mathbf{T}_A^{-T}(\phi_{d,e}) \mathbf{K}_P \tilde{\mathbf{x}}$$

- ★ in terms of elementary displacements

$$d\tilde{\mathbf{x}} = \dot{\tilde{\mathbf{x}}} \Big|_{\tilde{\mathbf{x}}=\mathbf{0}} dt = \mathbf{T}_A^{-1}(\mathbf{0})(\mathbf{v}_d^d - \mathbf{v}_e^d) dt = \mathbf{T}_A^{-1}(\mathbf{0}) d\mathbf{x}_{e,d}$$

⇓

$$\mathbf{h}_e = \mathbf{K}_P d\mathbf{x}_{e,d} \quad d\mathbf{x}_{e,d} = \mathbf{K}_P^{-1} \mathbf{h}_e$$

- Elastically compliant environment model

$$\mathbf{h}_e = \mathbf{K} d\mathbf{x}_{r,e}$$

$\mathbf{K} \equiv$ environment *stiffness*

- ★ positive semi-definite (only for the constrained motion directions)

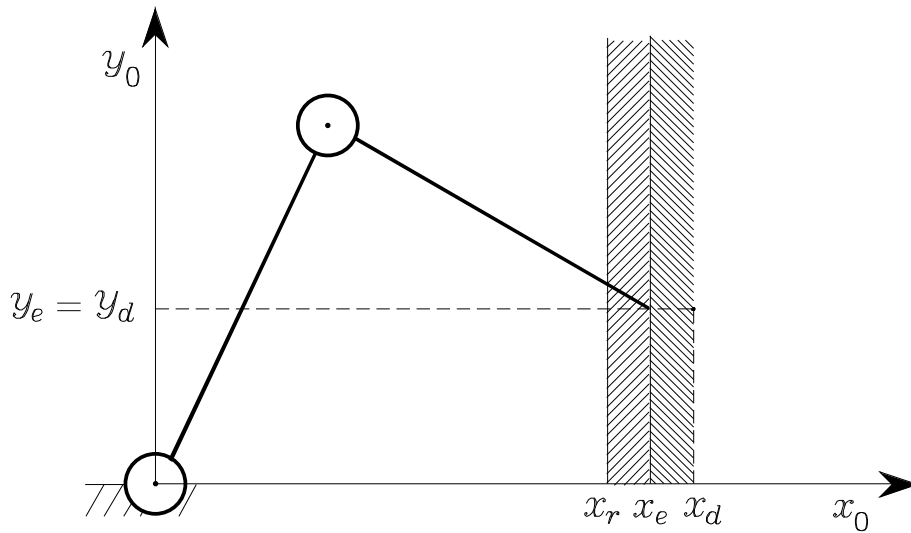
- *Active* vs. *passive* compliance . . . at equilibrium

$$\mathbf{h}_e = (\mathbf{I}_6 + \mathbf{K}\mathbf{K}_P^{-1})^{-1} \mathbf{K} d\mathbf{x}_{r,d}$$

$$d\mathbf{x}_{e,d} = \mathbf{K}_P^{-1} (\mathbf{I}_6 + \mathbf{K}\mathbf{K}_P^{-1})^{-1} \mathbf{K} d\mathbf{x}_{r,d}$$

- ★ manipulator \pm compliant
- ★ environment \pm compliant

Example



$$\mathbf{K} = \mathbf{K}_f = \text{diag}\{k_x, 0\}$$

$$\mathbf{K}_P = \text{diag}\{k_{Px}, k_{Py}\}$$

- At equilibrium

$$\mathbf{f}_e = \begin{bmatrix} \frac{k_{Px}k_x}{k_{Px} + k_x}(x_d - x_r) \\ 0 \end{bmatrix} \quad \mathbf{o}_e = \begin{bmatrix} \frac{k_{Px}x_d + k_x x_r}{k_{Px} + k_x} \\ y_d \end{bmatrix}$$

- ★ if $k_{Px}/k_x \gg 1$

$$x_e \approx x_d \quad f_x \approx k_x(x_d - x_r)$$

- ★ if $k_{Px}/k_x \ll 1$

$$x_e \approx x_r \quad f_{x\infty} \approx k_{Px}(x_d - x_r)$$

- Joint space active compliance

- ★ at equilibrium

$$\mathbf{K}_P \tilde{\mathbf{q}} = \mathbf{J}^T(\mathbf{q}) \mathbf{h}_e$$

$$\Downarrow$$

$$\tilde{\mathbf{q}} = \mathbf{K}_P^{-1} \mathbf{J}^T(\mathbf{q}) \mathbf{h}_e$$

$$d\tilde{\mathbf{x}} = \mathbf{J}(\mathbf{q}) \mathbf{K}_P^{-1} \mathbf{J}^T(\mathbf{q}) \mathbf{h}_e$$

IMPEDANCE CONTROL

- Operational space inverse dynamics control

$$\mathbf{u} = \mathbf{B}(\mathbf{q})\mathbf{y} + \mathbf{n}(\mathbf{q}, \dot{\mathbf{q}})$$

⇓

$$\ddot{\mathbf{q}} = \mathbf{y} - \mathbf{B}^{-1}(\mathbf{q})\mathbf{J}^T(\mathbf{q})\mathbf{h}_e$$

- ★ choice of new control input

$$\mathbf{y} = \mathbf{J}_A^{-1}(\mathbf{q})\mathbf{M}_d^{-1}(\mathbf{M}_d\ddot{\mathbf{x}}_d + \mathbf{K}_D\dot{\tilde{\mathbf{x}}} + \mathbf{K}_P\tilde{\mathbf{x}} - \mathbf{M}_d\dot{\mathbf{J}}_A(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}})$$

$$\mathbf{M}_d\ddot{\tilde{\mathbf{x}}} + \mathbf{K}_D\dot{\tilde{\mathbf{x}}} + \mathbf{K}_P\tilde{\mathbf{x}} = \mathbf{M}_d\mathbf{B}_A^{-1}(\mathbf{q})\mathbf{h}_A$$

$$\mathbf{B}_A(\mathbf{q}) = \mathbf{J}_A^{-T}(\mathbf{q})\mathbf{B}(\mathbf{q})\mathbf{J}_A^{-1}(\mathbf{q})$$

- Mechanical impedance
 - ★ mass \mathbf{M}_d
 - ★ damping \mathbf{K}_D
 - ★ stiffness \mathbf{K}_P

- Integration of contact *force measurements*

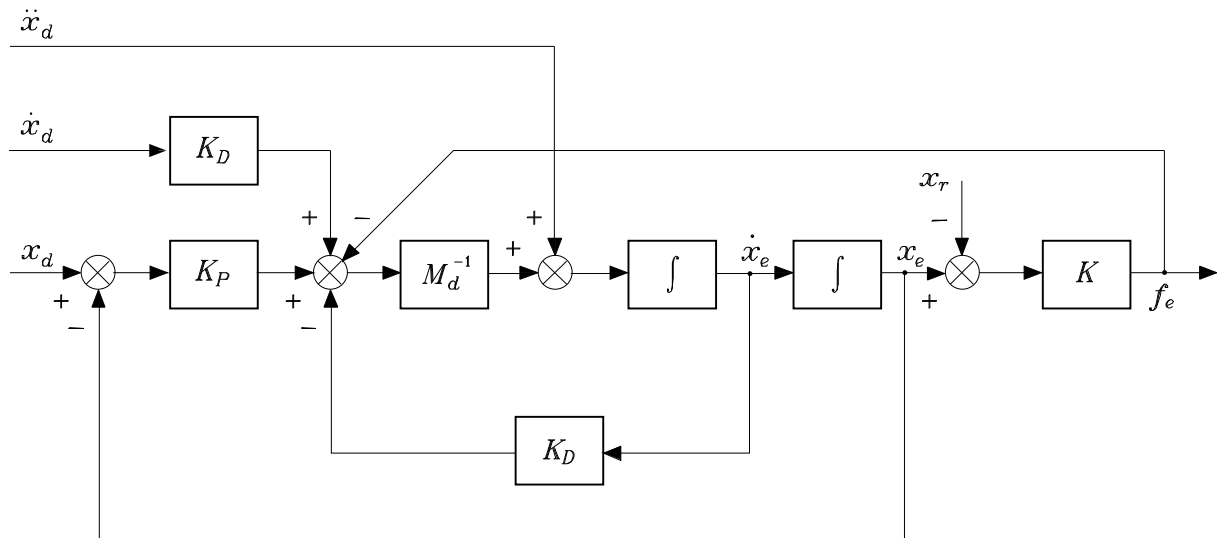
$$\mathbf{u} = \mathbf{B}(\mathbf{q})\mathbf{y} + \mathbf{n}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{J}^T(\mathbf{q})\mathbf{h}_e$$

$$\mathbf{y} = \mathbf{J}_A^{-1}(\mathbf{q})\mathbf{M}_d^{-1}(\mathbf{M}_d\ddot{\mathbf{x}}_d + \mathbf{K}_D\dot{\tilde{\mathbf{x}}} + \mathbf{K}_P\tilde{\mathbf{x}} - \mathbf{M}_d\dot{\mathbf{J}}_A(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} - \mathbf{h}_A)$$

⇓

$$\mathbf{M}_d\ddot{\tilde{\mathbf{x}}} + \mathbf{K}_D\dot{\tilde{\mathbf{x}}} + \mathbf{K}_P\tilde{\mathbf{x}} = \mathbf{h}_A$$

★ active *linear* impedance



- To avoid representation singularity problems . . .
 - ★ time-varying desired frame

$$\dot{\tilde{\mathbf{x}}} = -\mathbf{J}_{A_d}(\mathbf{q}, \tilde{\mathbf{x}})\dot{\mathbf{q}} + \mathbf{b}(\tilde{\mathbf{x}}, \mathbf{R}_d, \dot{\mathbf{o}}_d, \boldsymbol{\omega}_d)$$

$$\mathbf{b}(\tilde{\mathbf{x}}, \mathbf{R}_d, \dot{\mathbf{o}}_d, \boldsymbol{\omega}_d) = \begin{bmatrix} \mathbf{R}_d^T \dot{\mathbf{o}}_d + \mathbf{S}(\boldsymbol{\omega}_d^d) \mathbf{o}_{d,e}^d \\ \mathbf{T}^{-1}(\boldsymbol{\phi}_{d,e}) \boldsymbol{\omega}_d^d \end{bmatrix}$$

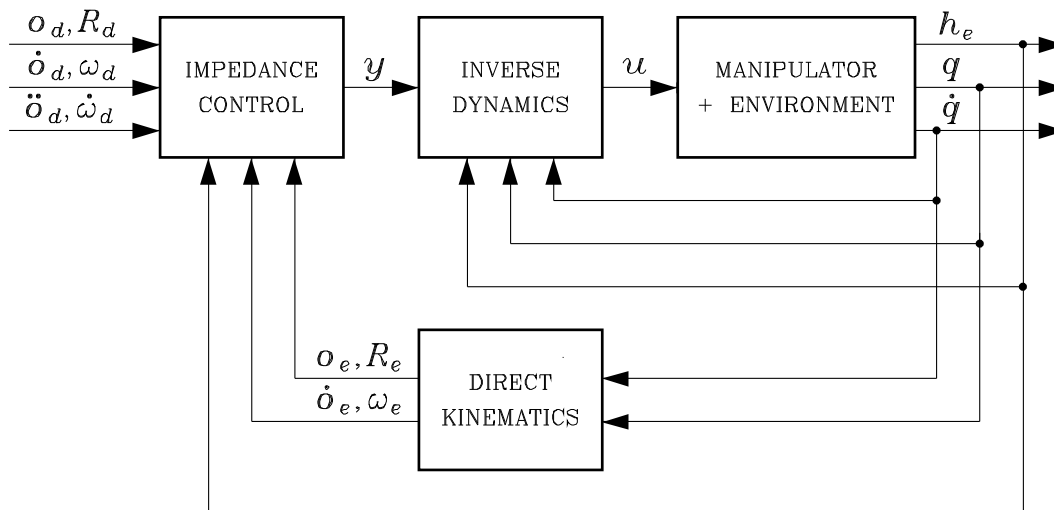
$$\ddot{\tilde{\mathbf{x}}} = -\mathbf{J}_{A_d} \ddot{\mathbf{q}} - \dot{\mathbf{J}}_{A_d} \dot{\mathbf{q}} + \dot{\mathbf{b}}$$

- ★ new control input

$$\mathbf{y} = \mathbf{J}_{A_d}^{-1} \mathbf{M}_d^{-1} (\mathbf{K}_D \dot{\tilde{\mathbf{x}}} + \mathbf{K}_P \tilde{\mathbf{x}} - \mathbf{M}_d \dot{\mathbf{J}}_{A_d} \dot{\mathbf{q}} + \mathbf{M}_d \dot{\mathbf{b}} - \mathbf{h}_e^d)$$

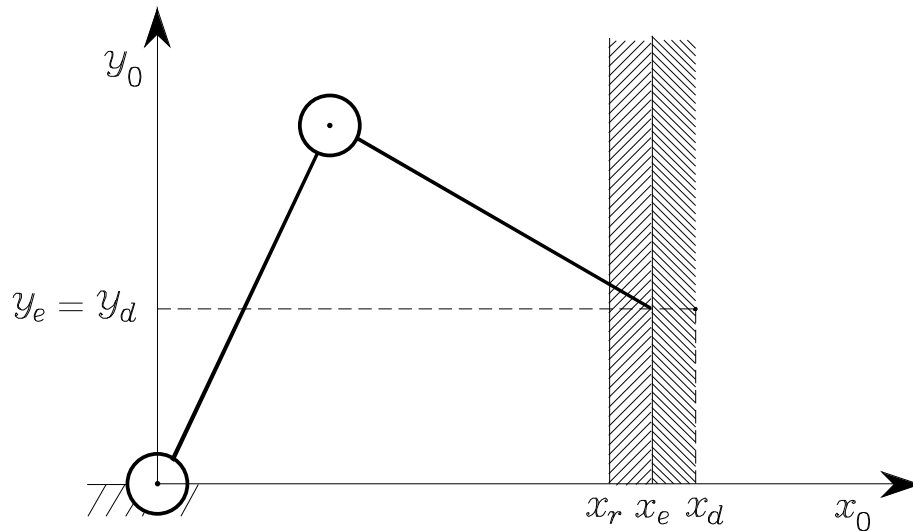
⇓

$$\mathbf{M}_d \ddot{\tilde{\mathbf{x}}} + \mathbf{K}_D \dot{\tilde{\mathbf{x}}} + \mathbf{K}_P \tilde{\mathbf{x}} = \mathbf{h}_e^d$$



- Passive impedance mechanical devices

Example



- Impedance control with force measurements

$$\mathbf{M}_d = \text{diag}\{m_{dx}, m_{dy}\}$$

$$\mathbf{K}_D = \text{diag}\{k_{Dx}, k_{Dy}\}$$

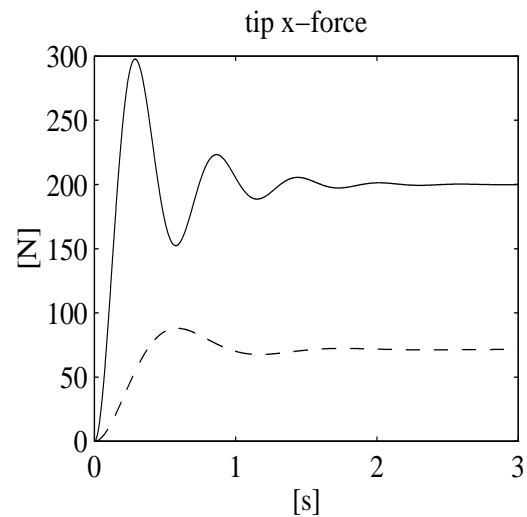
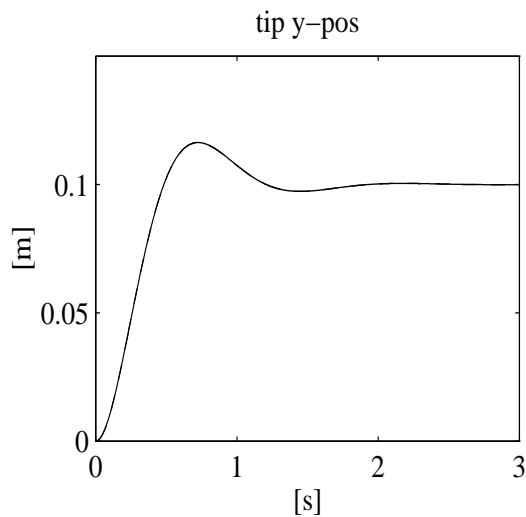
$$\mathbf{K}_P = \text{diag}\{k_{Px}, k_{Py}\}$$

- ★ manipulator–environment systems dynamics along the two operational space directions (constant x_d)

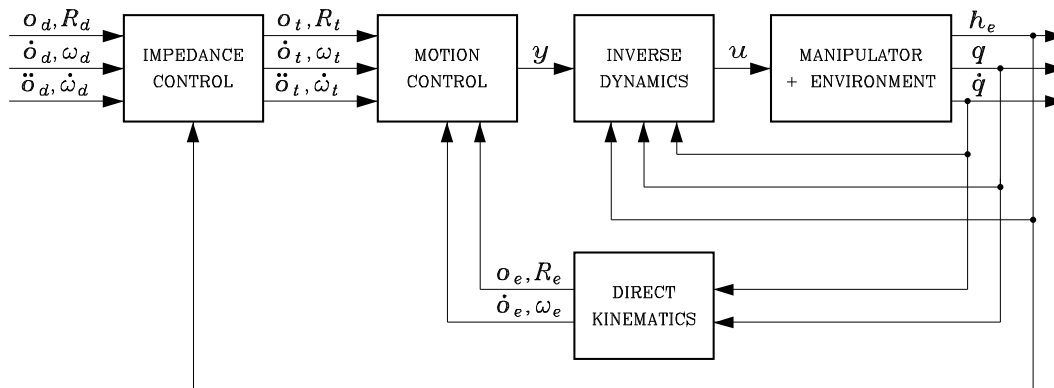
$$m_{dx}\ddot{x}_e + k_{Dx}\dot{x}_e + (k_{Px} + k_x)x_e = k_x x_r + k_{Px}x_d$$

$$m_{dy}\ddot{y}_e + k_{Dy}\dot{y}_e + k_{Py}y_e = k_{Py}y_d$$

- Environment ($x_r = 1$)
 - ★ more compliant $k_x = 10^3$ N/m
 - ★ less compliant $k_x = 10^4$ N/m
- Control
 - ★ $m_{dx} = m_{dy} = 100$
 - ★ $k_{Dx} = k_{Dy} = 500$
 - ★ $k_{Px} = k_{Py} = 2500$
- Manipulator
 - ★ $\mathbf{x}_e = [1 \ 0]^T$
 - ★ $\mathbf{x}_d = [1.1 \ 0.1]^T$



Admittance control



- Mechanical impedance between desired frame and *compliant frame t*

$$\mathbf{M}_t \ddot{\tilde{\mathbf{z}}} + \mathbf{K}_{Dt} \dot{\tilde{\mathbf{z}}} + \mathbf{K}_{Pt} \tilde{\mathbf{z}} = \mathbf{h}_e^d$$

- ★ compliant behaviour during interaction with environment
- ★ good robustness of inner motion control feedback loop (larger bandwidth)

FORCE CONTROL

- Compliance or impedance control \implies indirect force control
- Direct control
 - ★ PD control action on force (force measurements corrupted by noise)
 - ★ closure of *outer force regulation feedback loop* \implies reference input to inverse dynamics motion control scheme
- Hyp: only position variables
 - ★ elastic environment model

$$\mathbf{f}_e = \mathbf{K}(\mathbf{x}_e - \mathbf{x}_r)$$

Force control with inner position loop

$$\mathbf{u} = \mathbf{B}(\mathbf{q})\mathbf{y} + \mathbf{n}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{J}^T(\mathbf{q})\mathbf{h}_e$$

$$\mathbf{y} = \mathbf{J}^{-1}(\mathbf{q})\mathbf{M}_d^{-1}(-\mathbf{K}_D\dot{\mathbf{x}}_e + \mathbf{K}_P(\mathbf{x}_F - \mathbf{x}_e) - \mathbf{M}_d\dot{\mathbf{J}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}})$$

⇓

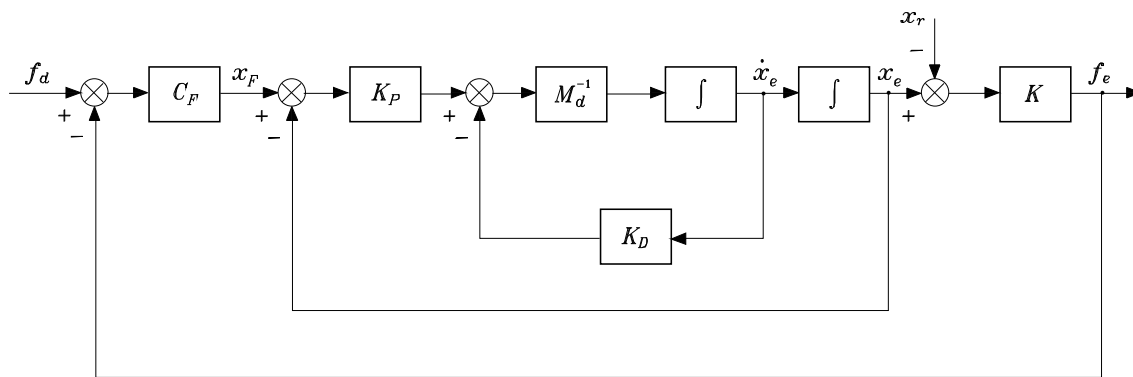
$$\mathbf{M}_d\ddot{\mathbf{x}}_e + \mathbf{K}_D\dot{\mathbf{x}}_e + \mathbf{K}_P\mathbf{x}_e = \mathbf{K}_P\mathbf{x}_F$$

- Choice of control

$$\mathbf{x}_F = \mathbf{C}_F(\mathbf{f}_d - \mathbf{f}_e)$$

⇓

$$\mathbf{M}_d\ddot{\mathbf{x}}_e + \mathbf{K}_D\dot{\mathbf{x}}_e + \mathbf{K}_P(\mathbf{I}_3 + \mathbf{C}_F\mathbf{K})\mathbf{x}_e = \mathbf{K}_P\mathbf{C}_F(\mathbf{K}\mathbf{x}_r + \mathbf{f}_d)$$



- PI action

$$\mathbf{C}_F = \mathbf{K}_F + \mathbf{K}_I \int^t (\cdot) d\zeta$$

★ choice of $\mathbf{K}_D, \mathbf{K}_P, \mathbf{K}_F, \mathbf{K}_I \implies$ stability margins + bandwidth

- At (stable) equilibrium

$$\mathbf{f}_e = \mathbf{f}_d$$

$$\mathbf{K} \mathbf{x}_e = \mathbf{K} \mathbf{x}_r + \mathbf{f}_d$$

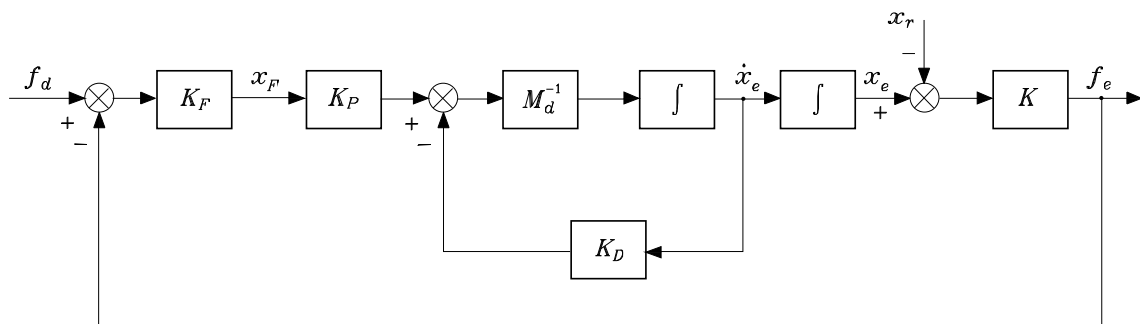
Force control with inner velocity loop

$$\mathbf{y} = \mathbf{J}^{-1}(\mathbf{q})\mathbf{M}_d^{-1}(-\mathbf{K}_D\dot{\mathbf{x}}_e + \mathbf{K}_P\mathbf{x}_F - \mathbf{M}_d\dot{\mathbf{J}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}})$$

$$\mathbf{x}_F = \mathbf{K}_F(\mathbf{f}_d - \mathbf{f}_e)$$

⇓

$$\mathbf{M}_d\ddot{\mathbf{x}}_e + \mathbf{K}_D\dot{\mathbf{x}}_e + \mathbf{K}_P\mathbf{K}_F\mathbf{K}\mathbf{x}_e = \mathbf{K}_P\mathbf{K}_F(\mathbf{K}\mathbf{x}_r + \mathbf{f}_d)$$



- At (stable) equilibrium

$$\mathbf{f}_e = \mathbf{f}_d$$

$$\mathbf{K}\mathbf{x}_e = \mathbf{K}\mathbf{x}_r + \mathbf{f}_d$$

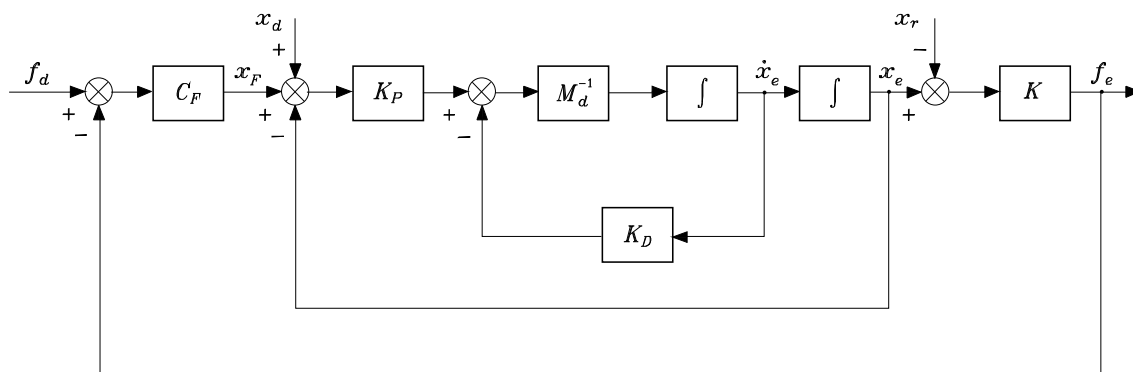
★ sensitive to unmodelled dynamics (lack of integral action)

Parallel force/position control

- * if $\mathbf{f}_d \notin \mathcal{R}(\mathbf{K}) \implies$ drift of \mathbf{x}_e
- * if $\mathbf{f}_d \in \mathcal{R}(\mathbf{K}) \implies \mathbf{x}_e$ (o $\dot{\mathbf{x}}_e$) $\rightarrow \mathbf{0}$

- Addition of \mathbf{x}_d

$$\mathbf{y} = \mathbf{J}^{-1}(\mathbf{q})\mathbf{M}_d^{-1}(-\mathbf{K}_D\dot{\mathbf{x}}_e + \mathbf{K}_P(\tilde{\mathbf{x}} + \mathbf{x}_F) - \mathbf{M}_d\dot{\mathbf{J}}_A(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}})$$

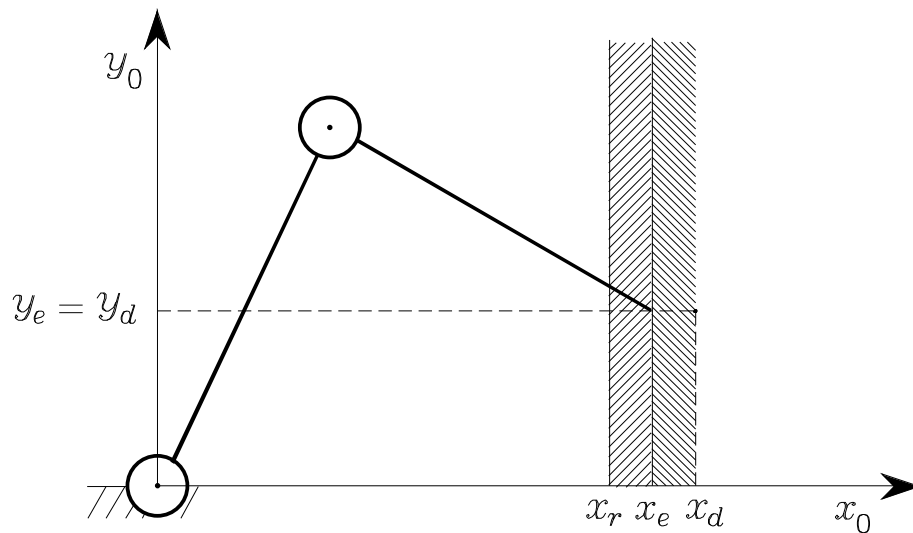


- At equilibrium

$$\mathbf{f}_e = \mathbf{f}_d$$

$$\mathbf{x}_e = \mathbf{x}_d + \mathbf{C}_F(\mathbf{K}(\mathbf{x}_r - \mathbf{x}_e) + \mathbf{f}_d)$$

Example



- Impedance (M_d, K_D, K_P) like above
- References $f_d = [10 \ 0]^T$ $x_d = [1.015 \ 0.1]^T$
- Force control

$$C_F = \text{diag}\{c_{Fx}, 0\}$$

1. control with inner position loop

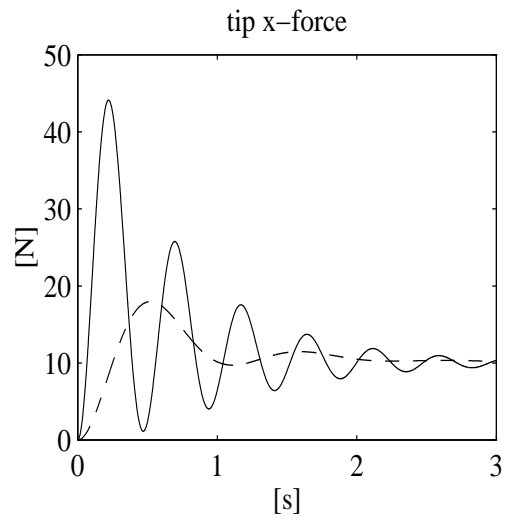
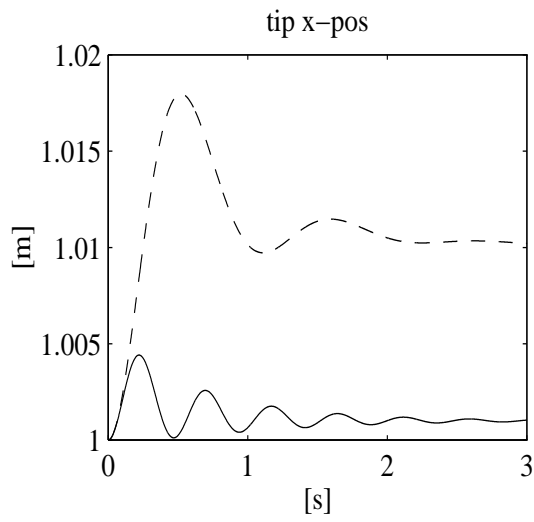
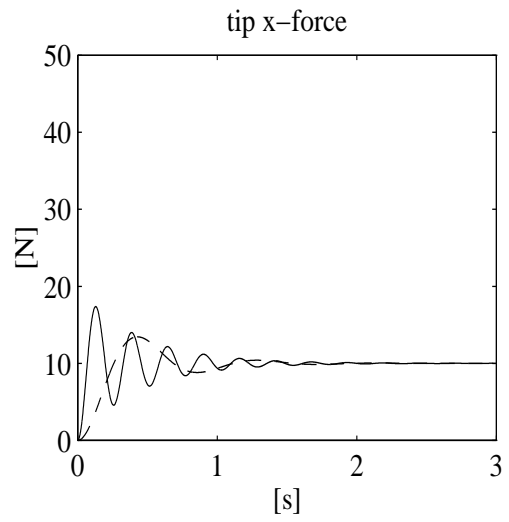
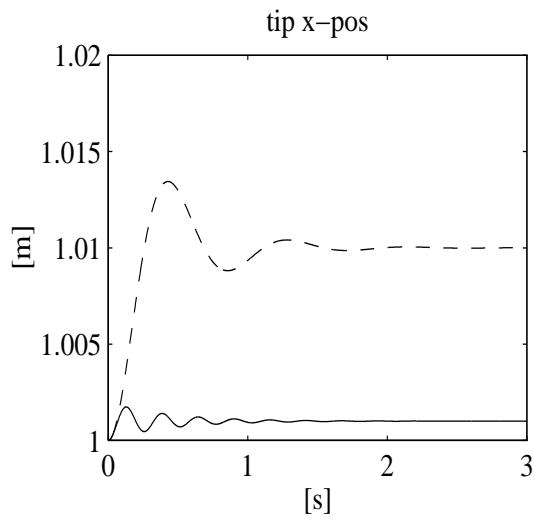
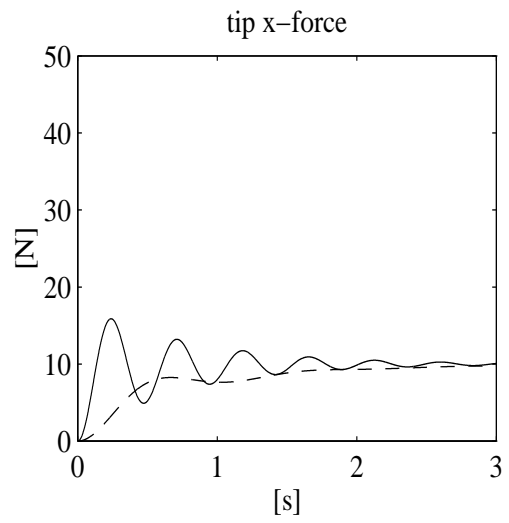
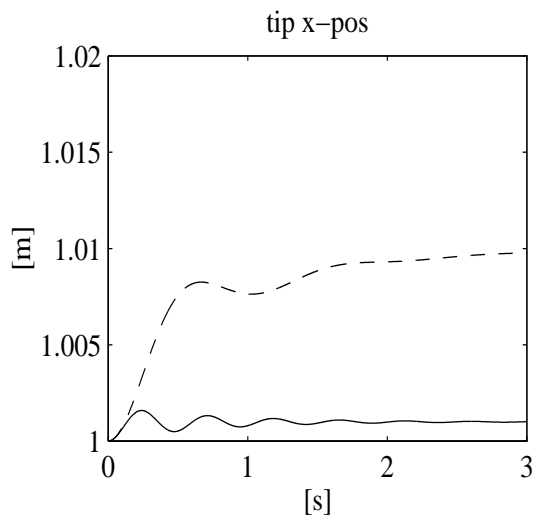
$$k_{Fx} = 0.00064 \quad k_{Ix} = 0.0016$$

2. control with inner velocity loop

$$k_{Fx} = 0.0024$$

3. parallel control

$$k_{Fx} = 0.00064 \quad k_{Ix} = 0.0016$$



CONSTRAINED MOTION

- Real contact situation
 - ★ kinematic constraints imposed by the environment on the end-effector motion
 - ★ during constrained motion, the end effector may exert dynamic forces and moments
 - ★ the contact force and moment may depend on the structural compliance of robot and force sensor
 - ★ local deformations of parts in contact (distributed contact areas), static and dynamic friction

- Simplifying assumptions for control
 - ★ perfectly rigid manipulator + environment, purely kinematic constraints
 - ★ perfectly rigid manipulator + compliance localized in environment

Rigid environment

- Constraint equations (holonomic constraints)

$$\varphi(\mathbf{q}) = \mathbf{0}$$

⇓

$$\mathbf{J}_\varphi(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0}$$

★ $\mathbf{J}_\varphi(\mathbf{q}) = \partial\varphi/\partial\mathbf{q}$: constraint Jacobian ($m \times 6$)

- In absence of friction

$$\mathbf{J}_\varphi(\mathbf{q})\delta\mathbf{q} = \mathbf{0}$$

★ principle of virtual works

$$\boldsymbol{\tau} = \mathbf{J}_\varphi^T(\mathbf{q})\boldsymbol{\lambda}$$

⇓ (contact kept with environment)

$$\mathbf{h}_e = \mathbf{J}^{-T}(\mathbf{q})\boldsymbol{\tau} = \mathbf{S}_f(\mathbf{q})\boldsymbol{\lambda}$$

$$\boldsymbol{\lambda} = \mathbf{S}_f^\dagger(\mathbf{q})\mathbf{h}_e$$

$$\mathbf{S}_f^\dagger = (\mathbf{S}_f^T \mathbf{W} \mathbf{S}_f)^{-1} \mathbf{S}_f^T \mathbf{W}$$

- Problem of invariance of solution

$$\mathbf{J}_\varphi(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0}$$

$$\mathbf{S}_f = \mathbf{J}^{-T}(\mathbf{q})\mathbf{J}_\varphi^T(\mathbf{q})$$

$$\Downarrow$$

$$\mathbf{J}_\varphi(\mathbf{q})\mathbf{J}^{-1}(\mathbf{q})\mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{S}_f^T \mathbf{v}_e = \mathbf{0}$$

- Reciprocity

$$\mathbf{h}_e^T \mathbf{v}_e = 0$$

- ★ $\mathcal{R}(\mathbf{S}_f) \equiv$ force control subspace

- ★ $\mathcal{R}(\mathbf{S}_v) \equiv$ velocity control subspace

$$\mathbf{v}_e = \mathbf{S}_v(\mathbf{q})\boldsymbol{\nu}$$

$$\mathbf{S}_f^T(\mathbf{q})\mathbf{S}_v(\mathbf{q}) = \mathbf{O}$$

$$\boldsymbol{\nu} = \mathbf{S}_v^\dagger(\mathbf{q})\mathbf{v}_e$$

- Selection matrices: $\mathbf{S}_f, \mathbf{S}_v, \mathbf{S}_f^\dagger, \mathbf{S}_v^\dagger$

- ★ projector in $\mathcal{R}(\mathbf{S}_f)$: $\mathbf{P}_f = \mathbf{S}_f\mathbf{S}_f^\dagger$

- ★ projector in $\mathcal{R}(\mathbf{S}_v)$: $\mathbf{P}_v = \mathbf{S}_v\mathbf{S}_v^\dagger$

Compliant environment

- Partially constrained motion
 - ★ environment modelled as pair of rigid bodies S and R

$$\mathbf{h}_e = \mathbf{S}_f \boldsymbol{\lambda}$$

$$d\mathbf{x}_{r,s} = \mathbf{C} \mathbf{h}_e$$

- ★ elementary displacement with respect to equilibrium pose

$$d\mathbf{x}_{r,e} = d\mathbf{x}_v + d\mathbf{x}_f$$

$$d\mathbf{x}_v = \mathbf{P}_v d\mathbf{x}_{r,e}$$

$$d\mathbf{x}_f = (\mathbf{I}_6 - \mathbf{P}_v) d\mathbf{x}_{r,e} = (\mathbf{I}_6 - \mathbf{P}_v) d\mathbf{x}_{r,s}$$

$$\mathbf{S}_f^T d\mathbf{x}_{r,e} = \mathbf{S}_f^T d\mathbf{x}_{r,s} = \mathbf{S}_f^T \mathbf{C} \mathbf{S}_f \boldsymbol{\lambda}$$

⇓

$$\mathbf{h}_e = \mathbf{K}' d\mathbf{x}_{r,e}$$

$$\mathbf{K}' = \mathbf{S}_f (\mathbf{S}_f^T \mathbf{C} \mathbf{S}_f)^{-1} \mathbf{S}_f^T$$

$$d\mathbf{x}_f = \mathbf{C}' \mathbf{h}_e$$

$$\mathbf{C}' = (\mathbf{I}_6 - \mathbf{P}_v) \mathbf{C}$$

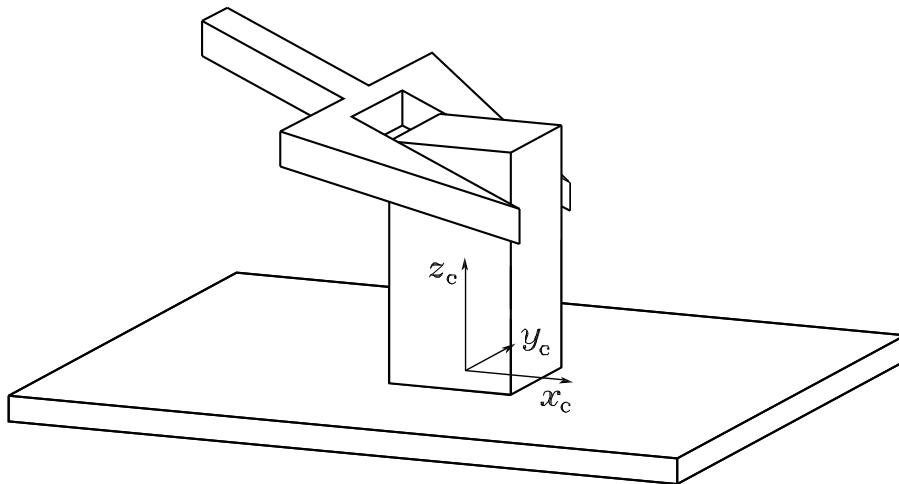
- ★ possibility of rigid contact along certain directions

NATURAL AND ARTIFICIAL CONSTRAINTS

- Force and velocity references chosen consistently with the environment
 - ★ it is not possible to impose both a force and a velocity along the same direction
- Kineto-static analysis of interaction
 - ★ *natural constraints* imposed by the environment along each task degree of freedom (velocity or force)
 - ★ *artificial constraints* subject to control along each task degree of freedom (force or velocity)
- Constraint frame $O_c-x_c y_c z_c$
 - ★ simplified task description in terms of natural and artificial constraints

Analysis of tasks

- Sliding on a planar surface



Natural Constraints	Artificial Constraints
\dot{o}_z^c	f_z^c
ω_x^c	μ_x^c
ω_y^c	μ_y^c
f_x^c	\dot{o}_x^c
f_y^c	\dot{o}_y^c
μ_z^c	ω_z^c

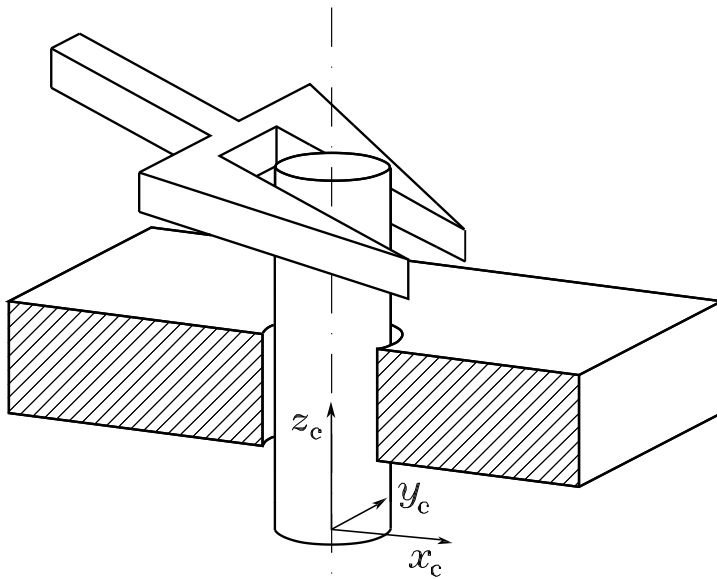
$$S_f = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$S_v = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

★ in case of elastically compliant plane (along z_c , about x_c , y_c)

$$\mathbf{K}'_m = \begin{bmatrix} c_{3,3} & c_{3,4} & c_{3,5} \\ c_{4,3} & c_{4,4} & c_{4,5} \\ c_{5,3} & c_{5,4} & c_{5,5} \end{bmatrix}^{-1}$$

- Peg-in-hole

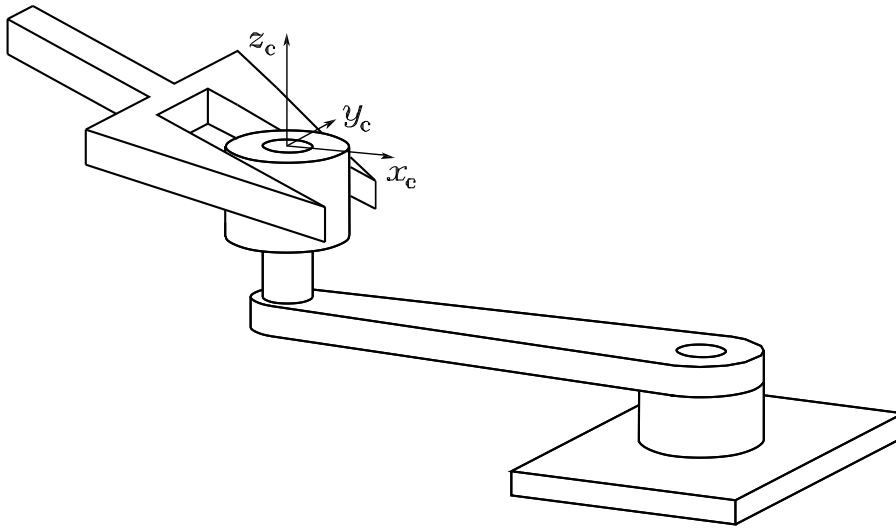


Natural Constraints	Artificial Constraints
\dot{o}_x^c	f_x^c
\dot{o}_y^c	f_y^c
ω_x^c	μ_x^c
ω_y^c	μ_y^c
f_z^c	\dot{o}_z^c
μ_z^c	ω_z^c

$$\mathbf{S}_f = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{S}_v = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

- Turning a crank



Natural Constraints	Artificial Constraints
\dot{o}_x^c	f_x^c
\dot{o}_z^c	f_z^c
ω_x^c	μ_x^c
ω_y^c	μ_y^c
f_y^c	\dot{o}_y^c
μ_z^c	ω_z^c

$$\mathbf{S}_f = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{S}_v = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

HYBRID FORCE/MOTION CONTROL

- Operational space dynamic model

$$\dot{v}_e = J(q)\ddot{q} + \dot{J}(q)\dot{q}$$

$$B_e(q)\dot{v}_e + n_e(q, \dot{q}) = \gamma_e - h_e$$

$$B_e = J^{-T} B J^{-1} \quad n_e = J^{-T} (C\dot{q} + g) - B_e \dot{J}\dot{q}$$

Compliant environment

$$d\mathbf{x}_{r,e} = P_v d\mathbf{x}_{r,e} + C' S_f \lambda.$$

↓ (fixed r frame)

$$\mathbf{v}_e = S_v \boldsymbol{\nu} + C' S_f \dot{\lambda}$$

$$\boldsymbol{\nu} = S_v^\dagger(q) \mathbf{v}_e \quad C' = (I_6 - P_v) C$$

★ common frame \equiv base frame, $\dot{S}_v = O$, $\dot{S}_f = O$, $\dot{C}' = O$

$$\dot{v}_e = S_v \dot{\boldsymbol{\nu}} + C' S_f \ddot{\lambda}$$

★ inverse dynamics control

$$\gamma_e = B_e(q)\alpha + n_e(q, \dot{q}) + h_e$$

$$\Downarrow$$

$$\dot{\mathbf{v}}_e = \boldsymbol{\alpha}$$

- *Hybrid force/velocity control*

$$\boldsymbol{\alpha} = \mathbf{S}_v \boldsymbol{\alpha}_v + \mathbf{C}' \mathbf{S}_f \mathbf{f}_\lambda$$

$$\Downarrow$$

$$\dot{\boldsymbol{\nu}} = \boldsymbol{\alpha}_v$$

$$\ddot{\boldsymbol{\lambda}} = \mathbf{f}_\lambda$$

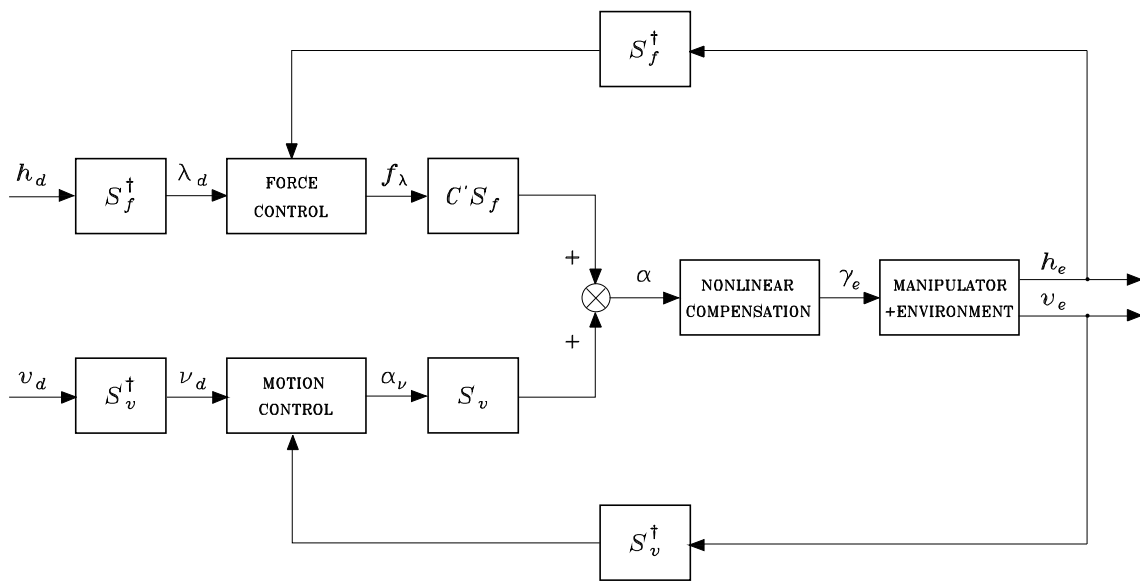
- ★ velocity control

$$\boldsymbol{\alpha}_v = \dot{\boldsymbol{\nu}}_d + \mathbf{K}_{P\nu}(\boldsymbol{\nu}_d - \boldsymbol{\nu}) + \mathbf{K}_{I\nu} \int_0^t (\boldsymbol{\nu}_d(\varsigma) - \boldsymbol{\nu}(\varsigma)) d\varsigma$$

- ★ force control

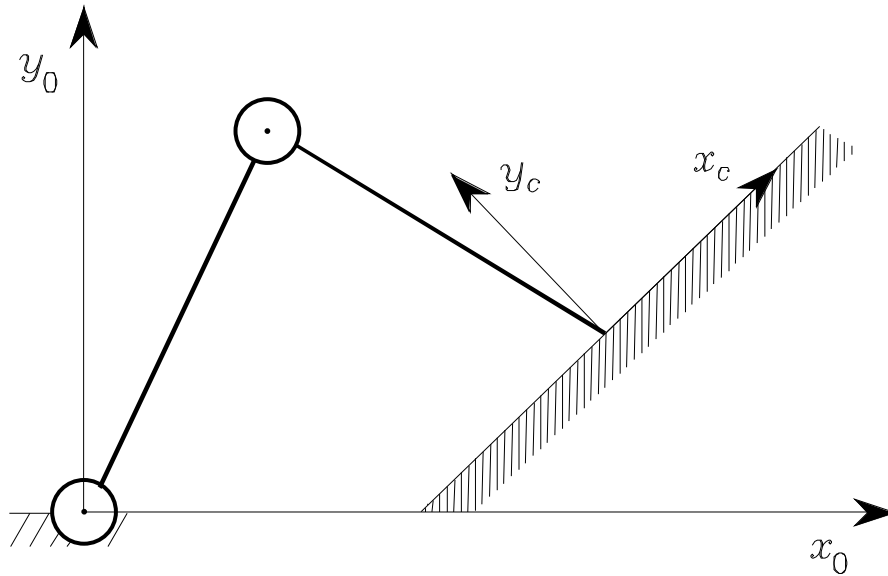
$$\mathbf{f}_\lambda = \ddot{\boldsymbol{\lambda}}_d + \mathbf{K}_{D\lambda}(\dot{\boldsymbol{\lambda}}_d - \dot{\boldsymbol{\lambda}}) + \mathbf{K}_{P\lambda}(\boldsymbol{\lambda}_d - \boldsymbol{\lambda})$$

$$\dot{\boldsymbol{\lambda}} = \mathbf{S}_f^\dagger \dot{\mathbf{h}}_e \quad \dots \quad \dot{\boldsymbol{\lambda}} = \mathbf{S}_f^\dagger \mathbf{K}' \mathbf{J}(\mathbf{q}) \dot{\mathbf{q}}$$



★ works also in case of available stiffness \hat{C}'

- Example



$$\mathbf{S}_f^c = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \mathbf{S}_v^c = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\mathbf{P}_f^c = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{P}_v^c = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{K}^{lc} = \begin{bmatrix} 0 & 0 \\ 0 & c_{2,2}^{-1} \end{bmatrix} \quad \mathbf{C}^{lc} = \begin{bmatrix} 0 & 0 \\ 0 & c_{2,2} \end{bmatrix}$$

$$\mathbf{R}_c = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$\mathbf{v}_e^c = \mathbf{S}_v^c \nu + \mathbf{C}^{lc} \mathbf{S}_f^c \dot{\lambda}$$

★ if $\mathbf{f}_e^c = [f_x^c \quad f_y^c]^T$, $\mathbf{v}_e^c = [\dot{o}_x^c \quad \dot{o}_y^c]^T$

★ $\nu = \dot{o}_x^c$, $\lambda = f_y^c$

★ in base frame

$$\mathbf{S}_f = \mathbf{R}_c \mathbf{S}_f^c = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \quad \mathbf{S}_v = \mathbf{R}_c \mathbf{S}_v^c = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$\mathbf{C}' = \mathbf{R}_c \mathbf{C}'^c \mathbf{R}_c^T = c_{2,2} \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

★ hybrid force/velocity control

$$\dot{\nu} = \ddot{o}_x^c = \alpha_\nu \quad \ddot{\lambda} = \ddot{f}_y^c = f_\lambda$$

Rigid environment

$$\mathbf{B}_e(\mathbf{q})\dot{\mathbf{v}}_e + \mathbf{n}_e(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\gamma}_e - \mathbf{h}_e$$

$$\mathbf{h}_e = \mathbf{S}_f \boldsymbol{\lambda}$$

⇓

$$\boldsymbol{\lambda} = \mathbf{B}_f(\mathbf{q}) \left(\mathbf{S}_f^T \mathbf{B}_e^{-1}(\mathbf{q}) (\boldsymbol{\gamma}_e - \mathbf{n}_e(\mathbf{q}, \dot{\mathbf{q}})) + \dot{\mathbf{S}}_f^T \mathbf{v}_e \right)$$

$$\mathbf{B}_f = (\mathbf{S}_f^T \mathbf{B}_e^{-1} \mathbf{S}_f)^{-1}$$

$$\mathbf{B}_e(\mathbf{q})\dot{\mathbf{v}}_e + \mathbf{S}_f \mathbf{B}_f(\mathbf{q}) \dot{\mathbf{S}}_f^T \mathbf{v}_e = \mathbf{P}(\mathbf{q}) (\boldsymbol{\gamma}_e - \mathbf{n}_e(\mathbf{q}, \dot{\mathbf{q}}))$$

$$\mathbf{P} = \mathbf{I}_6 - \mathbf{S}_f \mathbf{B}_f \mathbf{S}_f^T \mathbf{B}_e^{-1}$$

- *reduced order* dynamic model

$$\dot{\mathbf{v}}_e = \mathbf{S}_v \dot{\boldsymbol{\nu}} + \dot{\mathbf{S}}_v \boldsymbol{\nu}$$

- ★ $6 - m$ independent equations

$$\mathbf{B}_v(\mathbf{q}) \dot{\boldsymbol{\nu}} = \mathbf{S}_v^T \left(\boldsymbol{\gamma}_e - \mathbf{n}_e(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{B}_e(\mathbf{q}) \dot{\mathbf{S}}_v \boldsymbol{\nu} \right)$$

$$\boldsymbol{\lambda} = \mathbf{B}_f(\mathbf{q}) \mathbf{S}_f^T \mathbf{B}_e^{-1}(\mathbf{q}) \left(\boldsymbol{\gamma}_e - \mathbf{n}_e(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{B}_e(\mathbf{q}) \dot{\mathbf{S}}_v \boldsymbol{\nu} \right)$$

- *Hybrid force/velocity control*

$$\gamma_e = \mathbf{B}_e(\mathbf{q})\mathbf{S}_v\boldsymbol{\alpha}_v + \mathbf{S}_f\mathbf{f}_\lambda + \mathbf{n}_e(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{B}_e(\mathbf{q})\dot{\mathbf{S}}_v\boldsymbol{\nu}$$

⇓

$$\dot{\boldsymbol{\nu}} = \boldsymbol{\alpha}_v$$

$$\boldsymbol{\lambda} = \mathbf{f}_\lambda$$

- ★ force control

$$\mathbf{f}_\lambda = \boldsymbol{\lambda}_d + \mathbf{K}_{P\lambda}(\boldsymbol{\lambda}_d - \boldsymbol{\lambda})$$

$$\mathbf{f}_\lambda = \boldsymbol{\lambda}_d + \mathbf{K}_{I\lambda} \int_0^t (\boldsymbol{\lambda}_d(\varsigma) - \boldsymbol{\lambda}(\varsigma)) d\varsigma$$

- ★ position control

$$\boldsymbol{\alpha}_v = \ddot{\mathbf{r}}_d + \mathbf{K}_{Dv}(\dot{\mathbf{r}}_d - \boldsymbol{\nu}) + \mathbf{K}_{Pv}(\mathbf{r}_d - \mathbf{r})$$