Operational space control

- during interaction the environment sets constraints on the geometric paths
- successful execution (motion control) if the task is accurately planned
  - accurate model of both the robot manipulator and the environment
- a detailed description of the environment is difficult to obtain
  - planning errors may give rise to a contact force causing a deviation of the end-effector from the desired trajectory
  - the control system reacts to reduce such deviation leading to a build-up of the contact force until saturation of the joint actuators
- *contact force* is the quantity describing the state of interaction
- interaction control strategies
  - *indirect force control*
  - *direct force control*

Compliance Control

- contact forces arise naturally described in the operational space (*operational space control)*

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u} - \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{h}_e$$

- $\boldsymbol{h}_e \neq \boldsymbol{0}$ at the equilibrium it is

$$\boldsymbol{J}_A^T(\boldsymbol{q})\boldsymbol{K}_P\widetilde{\boldsymbol{x}} = \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{h}_e$$

- on the assumption of a full-rank Jacobian

$$\widetilde{\boldsymbol{x}} = \boldsymbol{K}_P^{-1}\boldsymbol{T}_A^T(\boldsymbol{x}_e)\boldsymbol{h}_e = \boldsymbol{K}_P^{-1}\boldsymbol{h}_A$$

- linear compliance (due to force components) is independent of the configuration, whereas torsional compliance (due to moment components) does depend on the current end-effector orientation through the matrix ***T***

Compliance Control

- if $\boldsymbol{h}_e \breve{\in} \mathcal{N}(\boldsymbol{J}^T)$, $\widetilde{\boldsymbol{x}} = \boldsymbol{0}$ with $\boldsymbol{h}_e \neq \boldsymbol{0}$ contact forces are completely balanced by the manipulator mechanical structure
- It is worth observing that the compliant (or stiff) behaviour of the manipulator is achieved by virtue of the control
  - *active compliance*
- *passive compliance* denotes mechanical systems with a prevalent dynamics of elastic type
- end-effector frame and desired $O_e\text{–}x_e y_e z_e$ $\quad O_d\text{–}x_d y_d z_d$

$$\boldsymbol{T}_e = \begin{bmatrix} \boldsymbol{R}_e & \boldsymbol{o}_e \\ \boldsymbol{0}^T & 1 \end{bmatrix} \qquad \boldsymbol{T}_d = \begin{bmatrix} \boldsymbol{R}_d & \boldsymbol{o}_d \\ \boldsymbol{0}^T & 1 \end{bmatrix}$$

$$\boldsymbol{T}_e^d = (\boldsymbol{T}_d)^{-1}\boldsymbol{T}_e = \begin{bmatrix} \boldsymbol{R}_e^d & \boldsymbol{o}_{d,e}^d \\ \boldsymbol{0}^T & 1 \end{bmatrix}$$

$$\boldsymbol{R}_e^d = \boldsymbol{R}_d^T \boldsymbol{R}_e \qquad \boldsymbol{o}_{d,e}^d = \boldsymbol{R}_d^T(\boldsymbol{o}_e - \boldsymbol{o}_d) \qquad \widetilde{\boldsymbol{x}} = -\begin{bmatrix} \boldsymbol{o}_{d,e}^d, \\ \boldsymbol{\phi}_{d,e} \end{bmatrix}$$

Analytic Jacobian – error in the operational space

$$\widetilde{\boldsymbol{x}} = \boldsymbol{K}_P^{-1}\boldsymbol{T}_A^T(\boldsymbol{x}_e)\boldsymbol{h}_e = \boldsymbol{K}_P^{-1}\boldsymbol{h}_A$$

$$\widetilde{\boldsymbol{x}} = -\begin{bmatrix} \boldsymbol{o}_{d,e}^{d}, \\ \boldsymbol{\phi}_{d,e} \end{bmatrix}$$

$$\dot{\widetilde{\boldsymbol{x}}} = -\boldsymbol{T}_A^{-1}(\boldsymbol{\phi}_{d,e})\begin{bmatrix} \boldsymbol{R}_d^T & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{R}_d^T \end{bmatrix}\boldsymbol{v}_e$$

$$\boldsymbol{v}_e = [\,\dot{\boldsymbol{o}}_e^T \quad \boldsymbol{\omega}_e^T\,]^T = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}}$$

$$\dot{\widetilde{\boldsymbol{x}}} = -\boldsymbol{J}_{A_d}(\boldsymbol{q}, \widetilde{\boldsymbol{x}})\dot{\boldsymbol{q}}$$

$$\boldsymbol{J}_{A_d}(\boldsymbol{q}, \widetilde{\boldsymbol{x}}) = \boldsymbol{T}_A^{-1}(\boldsymbol{\phi}_{d,e})\begin{bmatrix} \boldsymbol{R}_d^T & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{R}_d^T \end{bmatrix}\boldsymbol{J}(\boldsymbol{q})$$

PD control with gravity compensation

$$\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{J}_{A_d}^T(\boldsymbol{q}, \widetilde{\boldsymbol{x}})(\boldsymbol{K}_P \widetilde{\boldsymbol{x}} - \boldsymbol{K}_D \boldsymbol{J}_{A_d}(\boldsymbol{q}, \widetilde{\boldsymbol{x}})\dot{\boldsymbol{q}})$$

- in absence of interaction→ stability proof:

$$\boldsymbol{V}(\dot{\boldsymbol{q}}, \widetilde{\boldsymbol{x}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{B}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\widetilde{\boldsymbol{x}}^T \boldsymbol{K}_P \widetilde{\boldsymbol{x}} > 0 \qquad \forall \dot{\boldsymbol{q}}, \widetilde{\boldsymbol{x}} \neq \boldsymbol{0}$$

- in presence of interaction

$$\boldsymbol{J}_{A_d}^T(\boldsymbol{q})\boldsymbol{K}_P \widetilde{\boldsymbol{x}} = \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{h}_e$$

$$\boldsymbol{h}_e = \boldsymbol{T}_A^{-T}(\phi_{d,e})\boldsymbol{K}_P \widetilde{\boldsymbol{x}}$$

Elementary displacement

- in terms of elementary displacements

$$\boldsymbol{T}_A(\boldsymbol{0}) = \boldsymbol{I} \qquad \boldsymbol{h}_e = \boldsymbol{K}_P d\boldsymbol{x}_{e,d}$$

- environment

$$\boldsymbol{h}_e = \boldsymbol{K} d\boldsymbol{x}_{r,e}$$

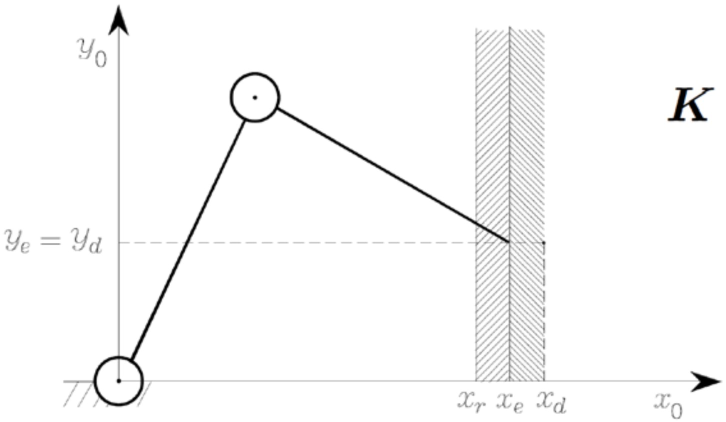
$$d\boldsymbol{x}_{r,e} = d\boldsymbol{x}_{r,d} - d\boldsymbol{x}_{e,d},$$

- equilibrium

$$\boldsymbol{h}_e = \left(\boldsymbol{I}_6 + \boldsymbol{K}\boldsymbol{K}_P^{-1}\right)^{-1} \boldsymbol{K} d\boldsymbol{x}_{r,d}$$

$$d\boldsymbol{x}_{e,d} = \boldsymbol{K}_P^{-1}\left(\boldsymbol{I}_6 + \boldsymbol{K}\boldsymbol{K}_P^{-1}\right)^{-1} \boldsymbol{K} d\boldsymbol{x}_{r,d}$$

Example



$$\boldsymbol{K} = \boldsymbol{K}_f = \mathrm{diag}\{k_x, 0\} \qquad \boldsymbol{K}_P = \mathrm{diag}\{k_{Px}, k_{Py}\}$$

$$\boldsymbol{f}_e = \begin{bmatrix} \dfrac{k_{Px}k_x}{k_{Px}+k_x}(x_d - x_r) \\ 0 \end{bmatrix} \qquad \boldsymbol{o}_e = \begin{bmatrix} \dfrac{k_{Px}x_d + k_x x_r}{k_{Px}+k_x} \\ y_d \end{bmatrix}$$

$k_{Px}/k_x \gg 1$

$$x_e \approx x_d \qquad f_x \approx k_x(x_d - x_r)$$

$k_{Px}/k_x \ll 1$

$$x_e \approx x_r \qquad f_{x\infty} \approx k_{Px}(x_d - x_r)$$

Active compliance in the joint space

$$\boldsymbol{K}_P \widetilde{\boldsymbol{q}} = \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{h}_e$$

$$\Downarrow$$

$$\widetilde{\boldsymbol{q}} = \boldsymbol{K}_P^{-1}\boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{h}_e$$

$$d\widetilde{\boldsymbol{x}} = \boldsymbol{J}(\boldsymbol{q})\boldsymbol{K}_P^{-1}\boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{h}_e$$

Impedance Control

- interaction of a manipulator with the environment under the action of an inverse dynamics control in the operational space

$$\boldsymbol{u} = \boldsymbol{B}(\boldsymbol{q})\boldsymbol{y} + \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}})$$

$$\Downarrow$$

$$\ddot{\boldsymbol{q}} = \boldsymbol{y} - \boldsymbol{B}^{-1}(\boldsymbol{q})\boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{h}_e$$

- nonlinear coupling term due to contact forces

$$\boldsymbol{y} = \boldsymbol{J}_A^{-1}(\boldsymbol{q})\boldsymbol{M}_d^{-1}\big(\boldsymbol{M}_d\ddot{\boldsymbol{x}}_d + \boldsymbol{K}_D\dot{\widetilde{\boldsymbol{x}}} + \boldsymbol{K}_P\widetilde{\boldsymbol{x}} - \boldsymbol{M}_d\dot{\boldsymbol{J}}_A(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}\big)$$

- mechanical impedance

$$\boldsymbol{M}_d\ddot{\widetilde{\boldsymbol{x}}} + \boldsymbol{K}_D\dot{\widetilde{\boldsymbol{x}}} + \boldsymbol{K}_P\widetilde{\boldsymbol{x}} = \boldsymbol{M}_d\boldsymbol{B}_A^{-1}(\boldsymbol{q})\boldsymbol{h}_A$$

$$\boldsymbol{B}_A(\boldsymbol{q}) = \boldsymbol{J}_A^{-T}(\boldsymbol{q})\boldsymbol{B}(\boldsymbol{q})\boldsymbol{J}_A^{-1}(\boldsymbol{q})$$

Impedance Control

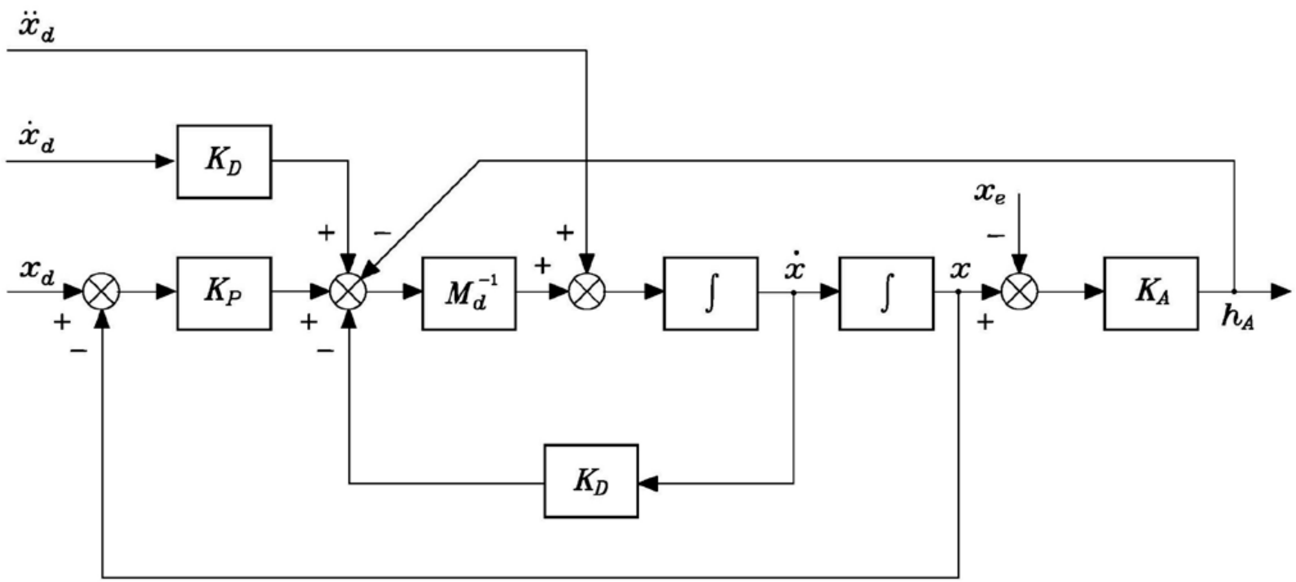
- to keep linearity and decoupling during interaction with the environment, it is then necessary to measure the contact force

$$\boldsymbol{u} = \boldsymbol{B}(\boldsymbol{q})\boldsymbol{y} + \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{h}_e$$

$$\boldsymbol{y} = \boldsymbol{J}_A^{-1}(\boldsymbol{q})\boldsymbol{M}_d^{-1}\big(\boldsymbol{M}_d\ddot{\boldsymbol{x}}_d + \boldsymbol{K}_D\dot{\widetilde{\boldsymbol{x}}} + \boldsymbol{K}_P\widetilde{\boldsymbol{x}} - \boldsymbol{M}_d\dot{\boldsymbol{J}}_A(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{h}_A\big)$$

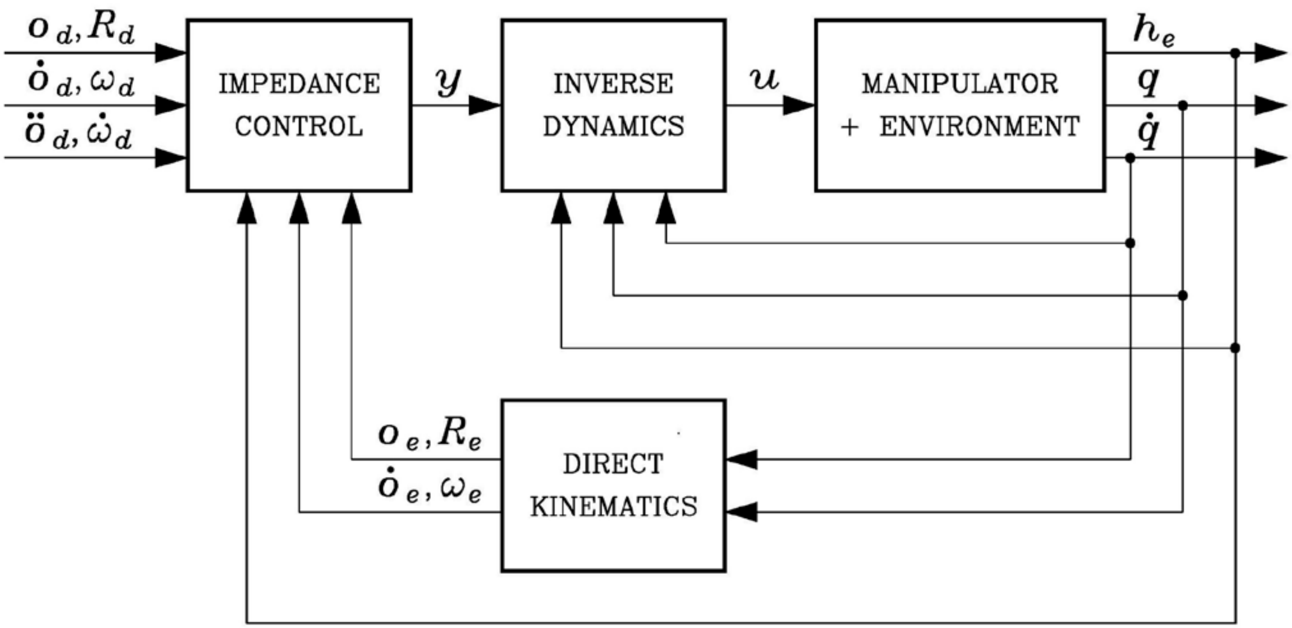
$$\Downarrow$$

$$\boldsymbol{M}_d\ddot{\widetilde{\boldsymbol{x}}} + \boldsymbol{K}_D\dot{\widetilde{\boldsymbol{x}}} + \boldsymbol{K}_P\widetilde{\boldsymbol{x}} = \boldsymbol{h}_A$$



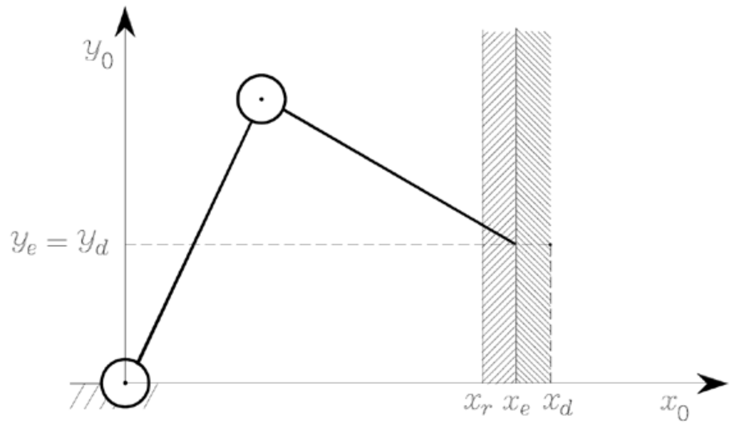
Impedance Control

- impedance depends on the current end-effector orientation through the matrix T
- to avoid this problem it is sufficient to redesign the control input y as a function of the operational space error



Impedance Control

- if the interaction force ***he*** is generated at the contact with an environment of proper mass, damping and stiffness, the system of manipulator with environment can be regarded as a mechanical system constituted by the parallel of the two impedances, and then its dynamic behaviour is conditioned by the relative weight between them



$$\boldsymbol{M}_d = \text{diag}\{m_{dx}, m_{dy}\}$$
$$\boldsymbol{K}_D = \text{diag}\{k_{Dx}, k_{Dy}\}$$
$$\boldsymbol{K}_P = \text{diag}\{k_{Px}, k_{Py}\}$$

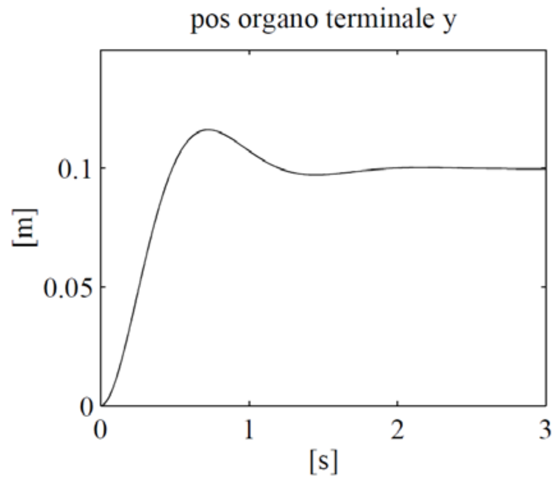
$$m_{dx}\ddot{x}_e + k_{Dx}\dot{x}_e + (k_{Px} + k_x)x_e = k_x x_r + k_{Px}x_d$$
$$m_{dy}\ddot{y}_e + k_{Dy}\dot{y}_e + k_{Py}y_e = k_{Py}y_d$$

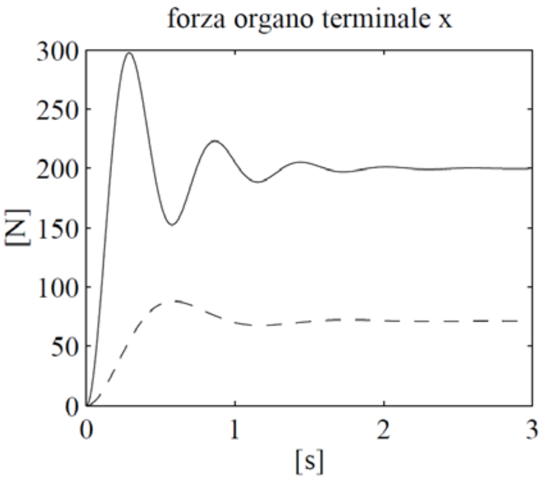
Example

$$m_{dx} = m_{dy} = 100$$

$$k_{Dx} = k_{Dy} = 500$$

$$k_{Px} = k_{Py} = 2500$$





$$k_x = 10^4 \text{ N/m}$$

$$k_x = 10^3 \text{ N/m}$$

Admittance Control

- the closed-loop behaviour is the more degraded by disturbances
- a possible solution is to separate the motion control from the impedance control
- the scheme is based on the concept of compliant frame, which is a suitable reference frame describing the ideal behaviour of the end-effector under impedance control
- this frame is specified by the position of the origin ***ot***, the rotation matrix ***Rt***, as well as by the liner and angular velocities and accelerations

$$\boldsymbol{M}_t \ddot{\tilde{\boldsymbol{z}}} + \boldsymbol{K}_{Dt} \dot{\tilde{\boldsymbol{z}}} + \boldsymbol{K}_{Pt} \tilde{\boldsymbol{z}} = \boldsymbol{h}_e^d$$