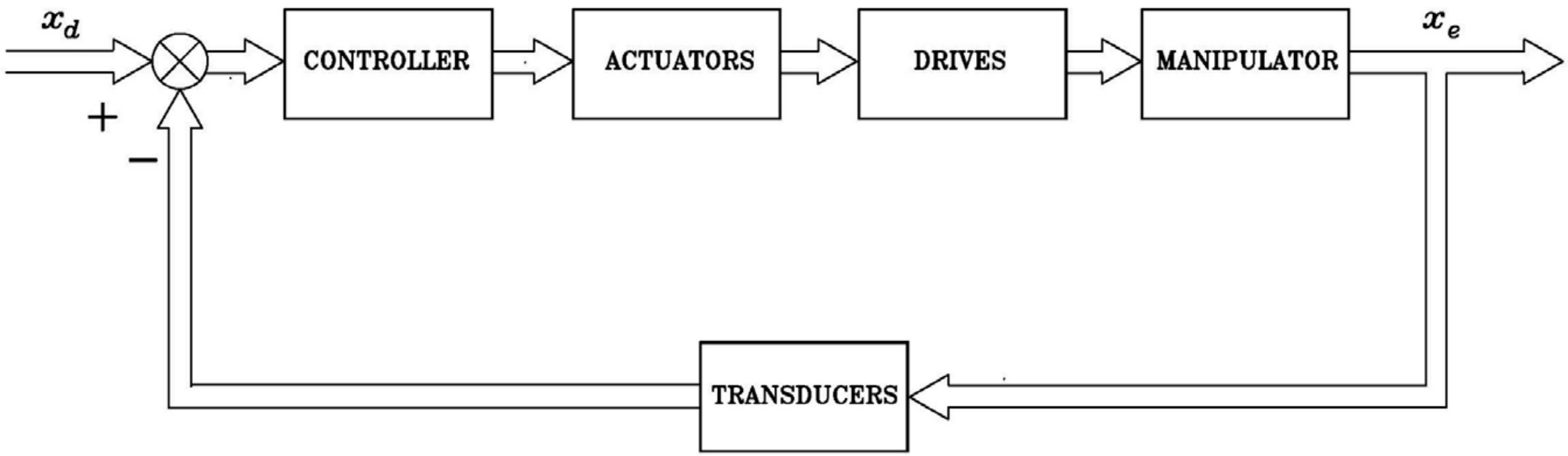
Motion control

- the technique may have a significant influence on the manipulator performance and on the possible range of applications
  - point-to-point control
  - trajectory tracking control in the operational space
- mechanical design has an influence on the control scheme
  - Cartesian manipulator
  - anthropomorphic manipulator
- driving system of the joints
  - electric motors with reduction gears of high ratios
    - tends to linearize system dynamics (to decouple the joints in view of the reduction of nonlinearity effects)
    - but joint friction, elasticity and backlash may limit system performance
  - a robot actuated with direct drives
    - eliminates the drawbacks due to friction, elasticity and backlash
    - but the weight of nonlinearities and couplings between the joints becomes relevant

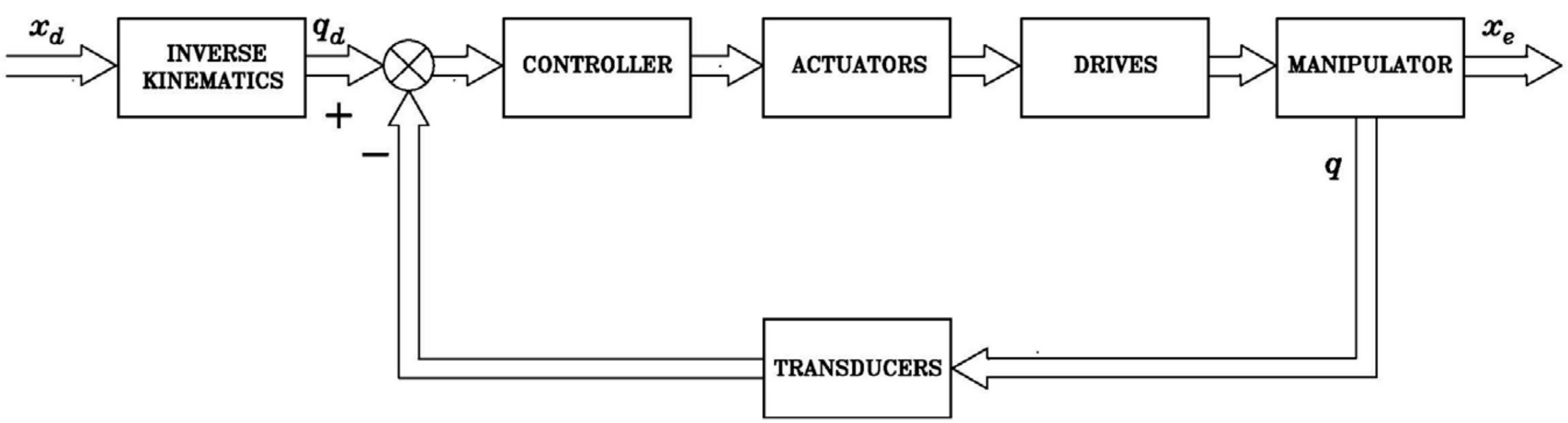
Operational space control

- *operational space control* problem follows a global approach that requires greater algorithmic complexity
- inverse kinematics is now embedded into the feedback control loop
- conceptual advantage regards the possibility of acting directly on operational space variables
  - measurement of operational space variables is often performed not directly, but through the evaluation of direct kinematics functions starting from measured joint space variables



Joint space control

- is actually articulated in two subproblems
  - manipulator inverse kinematics is solved to transform the motion requirements
  - joint space control scheme is designed that allows the actual motion $\boldsymbol{q}$ to track the reference inputs
- drawback: a joint space control scheme does not influence the operational space variables $\boldsymbol{x}_e$ which are controlled in an open-loop fashion



Centralized Control

- is based on the (partial or complete) knowledge of the manipulator dynamic model
- the robot is a multivariable system with *n* inputs (joint torques) and *n* outputs (joint positions) interacting between them by means of nonlinear relations
- *nonlinear centralized control* laws needed for high manipulator dynamic performance

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u}$$

PD control with gravity compensation

- assigned a *constant* equilibrium posture as the vector of ***q****d (point-to-point motion)*
- Lyapunov direct method to determine the control input which stabilizes the system around the equilibrium posture
- State $[\widetilde{\boldsymbol{q}}^T \quad \dot{\boldsymbol{q}}^T]^T$ $\qquad \widetilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$
- Lyapunov function candidate

$$V(\dot{\boldsymbol{q}}, \widetilde{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{B}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\widetilde{\boldsymbol{q}}^T \boldsymbol{K}_P \widetilde{\boldsymbol{q}} > 0 \qquad \forall \dot{\boldsymbol{q}}, \widetilde{\boldsymbol{q}} \neq \boldsymbol{0}$$

$$\dot{V} = \dot{\boldsymbol{q}}^T \boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T \dot{\boldsymbol{B}}(\boldsymbol{q})\dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}^T \boldsymbol{K}_P \widetilde{\boldsymbol{q}}$$

$$= \frac{1}{2}\dot{\boldsymbol{q}}^T \left(\dot{\boldsymbol{B}}(\boldsymbol{q}) - 2\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}^T \boldsymbol{F} \dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T \left(\boldsymbol{u} - \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{K}_P \widetilde{\boldsymbol{q}}\right)$$

PD control with gravity compensation

- control choice

$$\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{K}_P\widetilde{\boldsymbol{q}} - \boldsymbol{K}_D\dot{\boldsymbol{q}}$$

$$\Downarrow$$

$$\dot{V} = -\dot{\boldsymbol{q}}^T(\boldsymbol{F} + \boldsymbol{K}_D)\dot{\boldsymbol{q}}$$

$$\dot{V} = 0 \qquad\qquad \dot{\boldsymbol{q}} = \boldsymbol{0}, \forall\widetilde{\boldsymbol{q}}$$

- the derivative term causes an increase of the absolute values of $\dot{V}$ along the system trajectories
  - crucial when direct-drive manipulators are considered

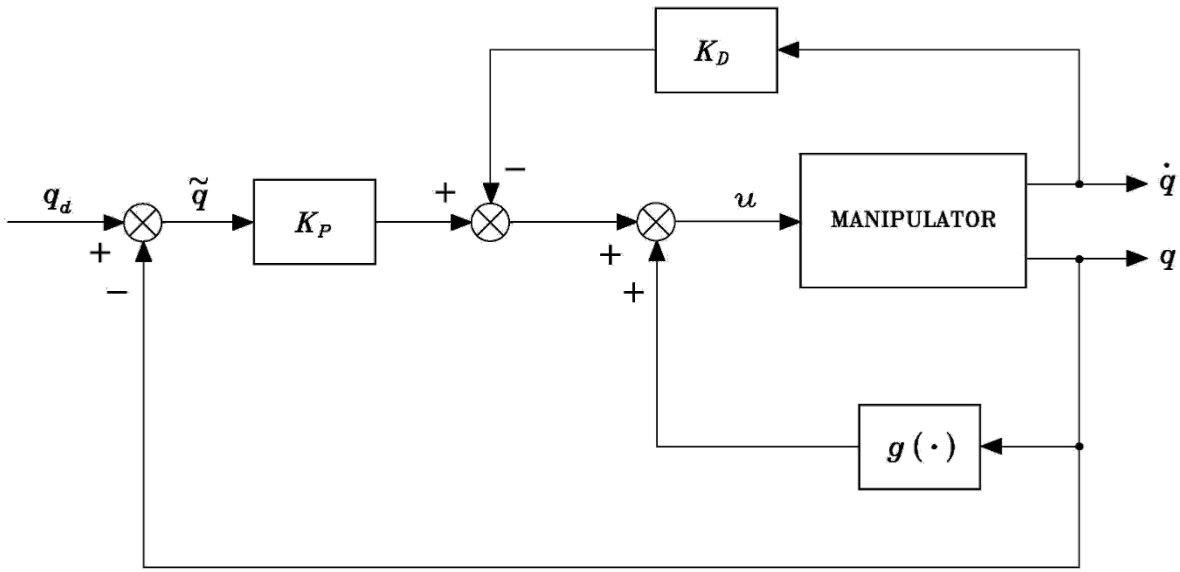
PD control with gravity compensation

- dynamics of the controlled system

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = g(q) + K_P\tilde{q} - K_D\dot{q}$$

- at the equilibrium $(\dot{q} \equiv \ddot{q} \equiv 0)$

$$K_P\tilde{q} = 0 \qquad \Longrightarrow \qquad \tilde{q} = q_d - q \equiv 0$$



- if $g(q)$ is not well compensated?

Inverse Dynamics Control

- tracking a joint space trajectory
- dynamic model

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{u}$$

$$\boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q})$$

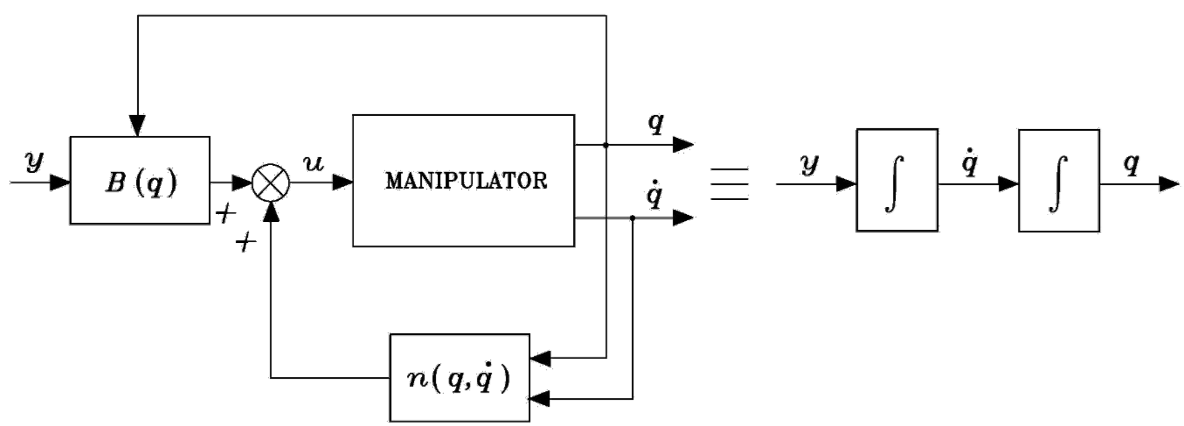
- an *exact linearization* of system dynamics obtained by means of a *nonlinear state feedback*

$$\forall \boldsymbol{q}$$

$$\boldsymbol{u} = \boldsymbol{B}(\boldsymbol{q})\boldsymbol{y} + \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}})$$

$$\Downarrow$$

$$\ddot{\boldsymbol{q}} = \boldsymbol{y}$$



Inverse Dynamics Control

- the manipulator control problem is reduced to that of finding a stabilizing control law ***y***

$$\boldsymbol{y} = -\boldsymbol{K}_P \boldsymbol{q} - \boldsymbol{K}_D \dot{\boldsymbol{q}} + \boldsymbol{r}$$

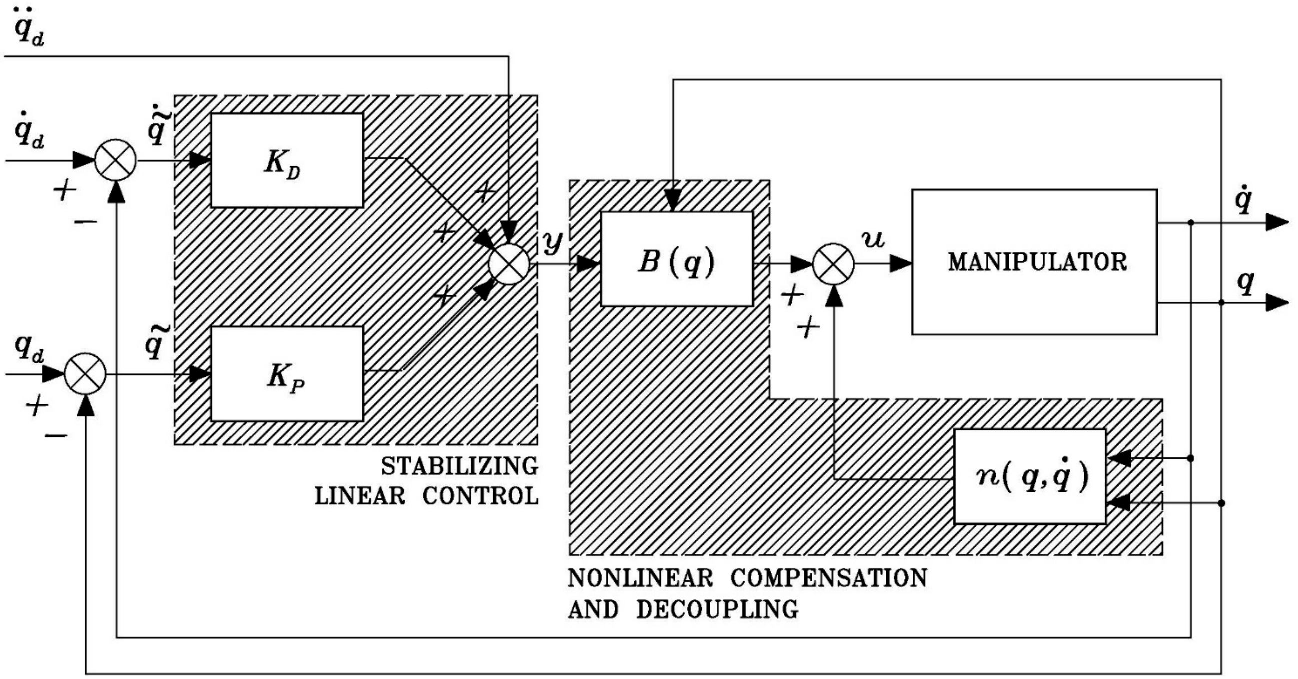
$$\boldsymbol{r} = \ddot{\boldsymbol{q}}_d + \boldsymbol{K}_D \dot{\boldsymbol{q}}_d + \boldsymbol{K}_P \boldsymbol{q}_d$$

$$\Downarrow$$

$$\ddot{\tilde{\boldsymbol{q}}} + \boldsymbol{K}_D \dot{\tilde{\boldsymbol{q}}} + \boldsymbol{K}_P \tilde{\boldsymbol{q}} = \boldsymbol{0}$$

Inverse Dynamics Control

- the *inner loop* is to obtain a *linear and decoupled input/output relationship*
- the *outer loop* is required to *stabilize the overall system*



- this technique is based on the assumption of perfect cancellation of dynamic terms
- inverse dynamics computation performed at sampling times ~ a millisecond
- severe constraints on the hardware/software architecture of the control system