Epipolar Geometry and Visual Servoing

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joint with with

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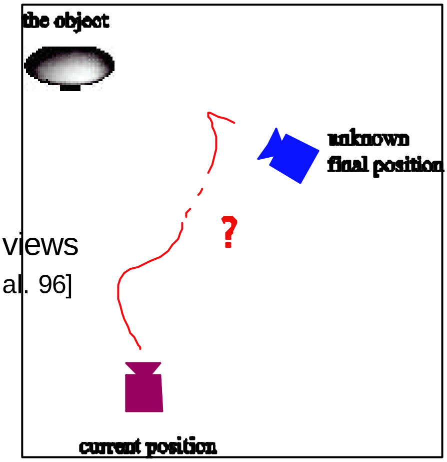
Bertinoro (FC), 14–16 Luglio 2003

Slides' Version — June 9, 2003

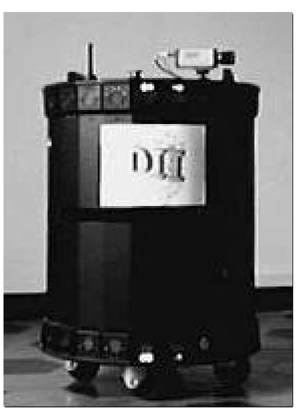
This is a preliminary version of my talk. An update version of the slides will be soon available at the School Web Site or at my home page.

Visual servoing problem



- Unknown desired position
- Available current and desired views
- Image based VS [Hutchinson et al. 96]

Fully actuated camera-robot

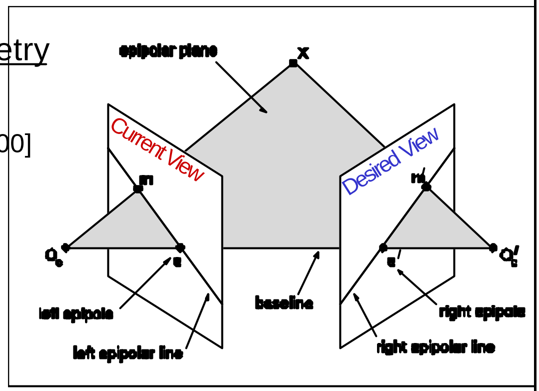


Contribution

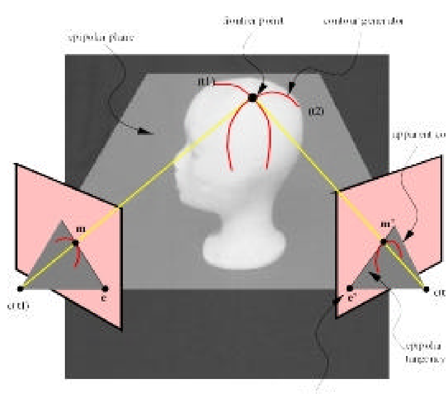
<u>Two-views Epipolar Geometry</u>

fundamental matrix *F*

[Faugeras 1993, Zissermann 2000]

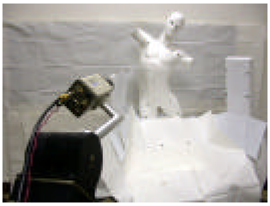


Natural setting for contour based VS



Epipolar tangency for surface reconstruction applications [Cipolla 2000]. Contours are very important in unstructured environments and in outdoor navigation. No contour parameterization is required.

Initial and final positions



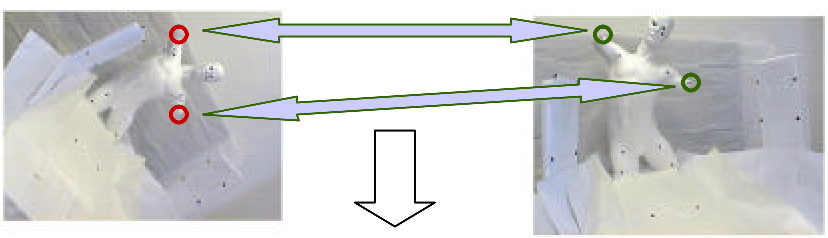
Current position

Current image

Target position

Target image

The **Fundamental Matrix F** is estimated from the correspondences between current and target images.



Fundamental Matrix F

Current Epipole

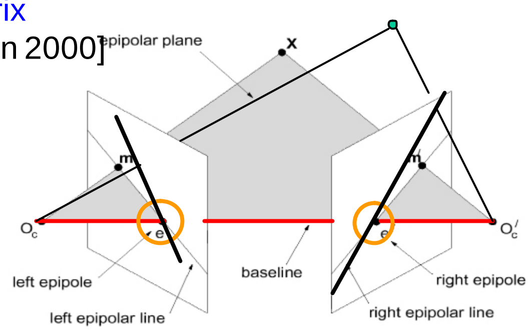
Desired Epipole

Given a set of corresponding points m_i', m_i in homogeneous coordinates, there exists a matrix $F \in \mathcal{R}^{3\times3}$, called the fundamental matrix [Faugeras 1993, Zissermann 2000] such that:

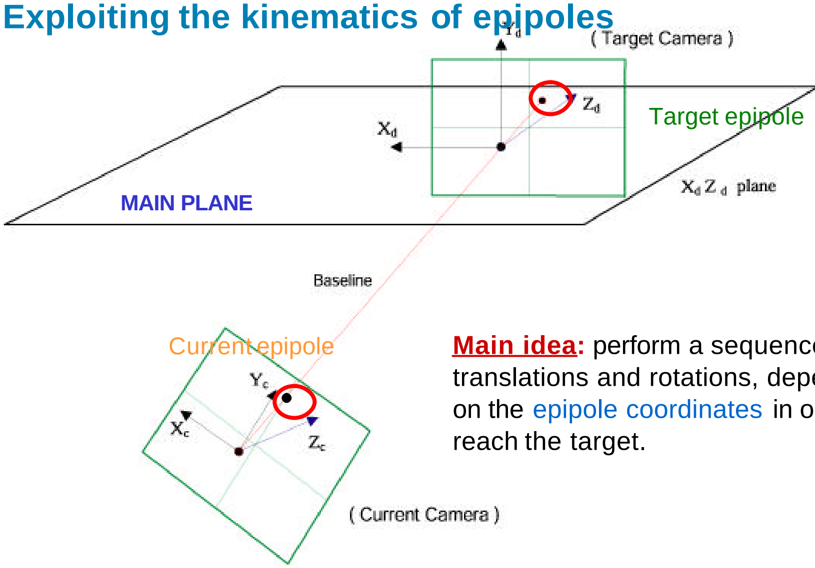
$$m_i^T F m_i = 0$$

$$e = \ker(F)$$

$$e' = \ker(F^T)$$



Exploiting the kinematics of epipoles



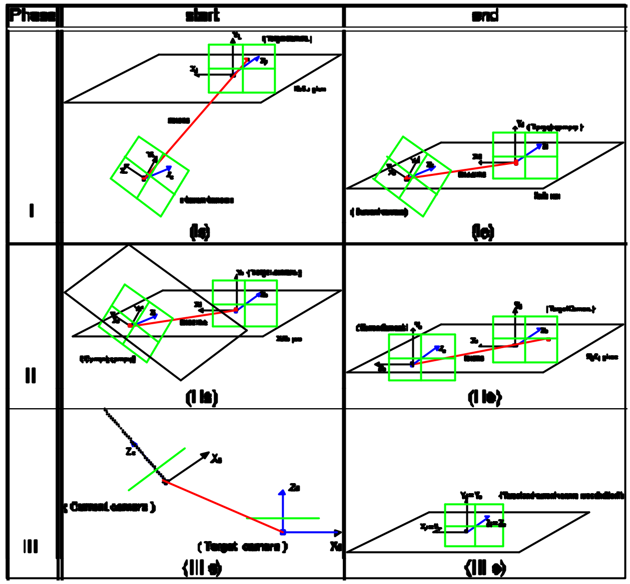
Main idea: perform a sequence of translations and rotations, depending on the epipole coordinates in order to reach the target.

3-Phases Algorithm

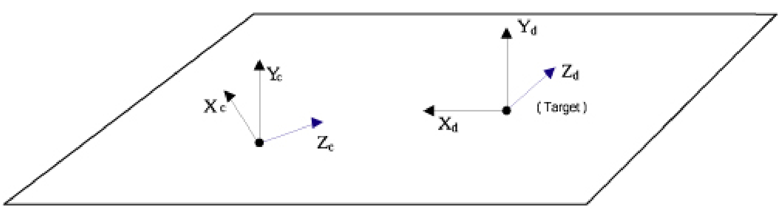
Reaching the target plane

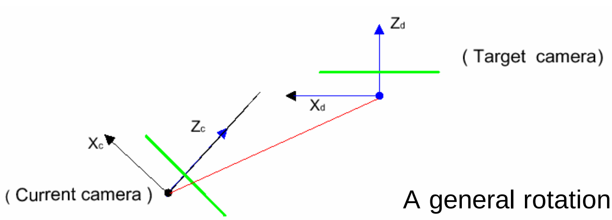
Getting main target coplanarity

the Planar Case



the Planar Case or Planar Homing





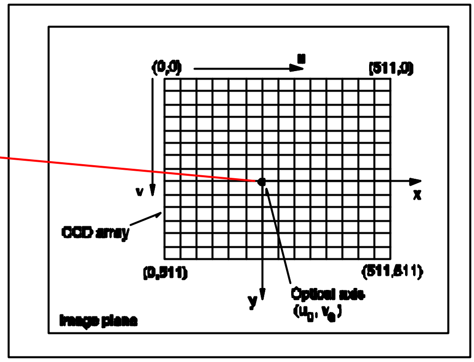
A general rotation **R** and translation **t**
Exists between current and target frames

Top view

Essential and Fudamental Matrices

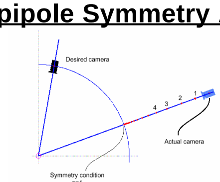
$$F = K^{-T} E K^{-1}$$
$$E = [t]_{\times} R$$

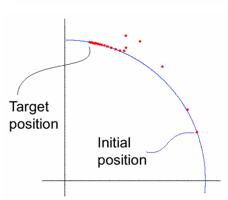
$$K = \begin{bmatrix} f_x & 0 & u_0 \\ 0 & f_y & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$



two planar Homing methods

Epipole Symmetry Algorithm

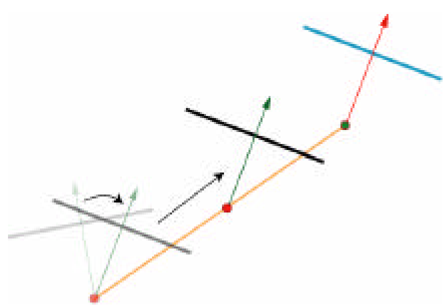




[ICRA 2001]
partially calibrated camera
principal point (u_0,v_0)

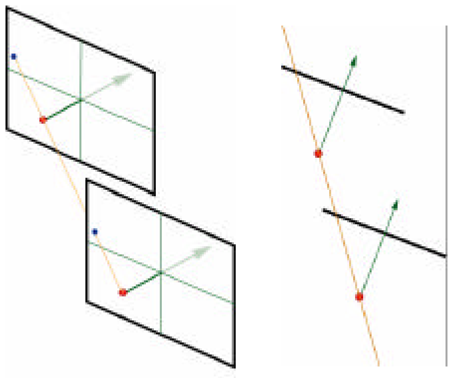
Auto-Epipole Algorithm

Rotate to get pure translation !



uncalibrated camera

Fundamental matrix under pure translation



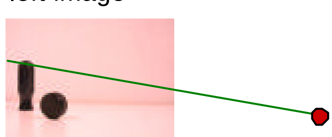
$$R = I$$
$$\Downarrow$$
$$E = [t]_\times I = [t]_\times$$
$$F = [Kt]_\times$$
$$\Downarrow$$
$$\ker(F) = \ker(F^\top) = Kt$$

Theory not only for planar cases !

the auto-epipole

What does $\ker(F) = \ker(F^\top)$ implies ?

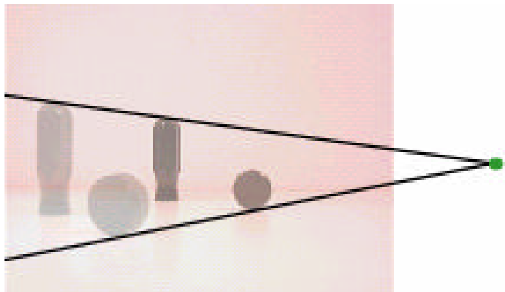
left image



right image

- Current and desired epipoles are equal
- Epipolar lines have the same angular coeff.

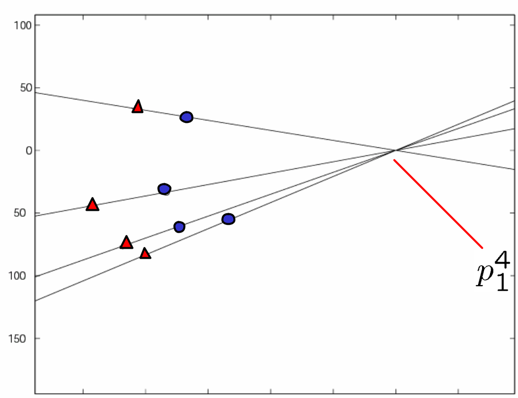
Overlapped images



If the camera motion is pure translation, a bi-tangent
to the corresponding apparent contours coincides with an epipolar line.
The epipole is the intersection of the bi-tangents.

Pure traslation

Four bi-tangents obtained from four corresponding points in the overlapped images

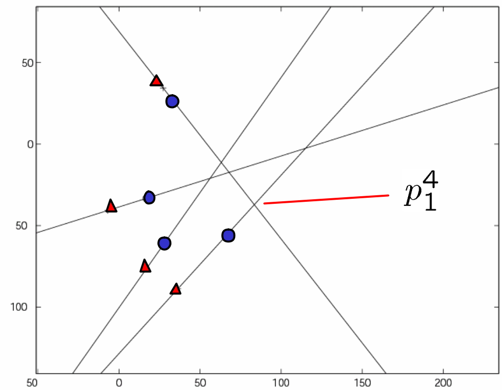


$$R = I$$

$$p_1^2 = p_1^3 = p_1^4 = p_2^3 = p_2^4 = p_3^4$$

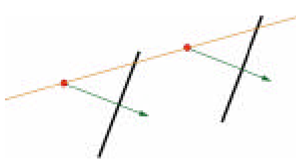
Non pure translation

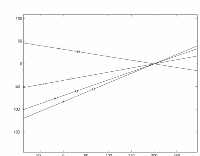
Four bi-tangents obtained from four corresponding points in the overlapped images



$$R \neq I$$

$$p_1^2 \neq p_1^3 \neq p_1^4 \neq p_2^3 \neq p_2^4 \neq p_3^4$$





Same orientation (pure translation) ⇒ **All the bi-tangents intersect in the same point (the autoepipole)**

⇐ ?

Theorem

Given 8 point matches (u_a,u_d) in the two views and let the rank of matrix A be 8, then when all the bi-tangents intersect in the same point the two cameras have the same orientation and the point intersection is the epipole.

$$A = \begin{pmatrix} (u_{a_1}u_{d_1}, u_{a_1}v_{d_1}, u_{a_1}, v_{a_1}u_{d_1}, v_{a_1}v_{d_1}, v_{a_1}, u_{d_1}, v_{d_1}, 1) \\ \ldots\ldots\ldots \qquad\qquad \ldots\ldots\ldots \\ (u_{a_8}u_{d_8}, u_{a_8}v_{d_8}, u_{a_8}, v_{a_8}u_{d_8}, v_{a_8}v_{d_8}, v_{a_8}, u_{d_8}, v_{d_8}, 1) \end{pmatrix}$$

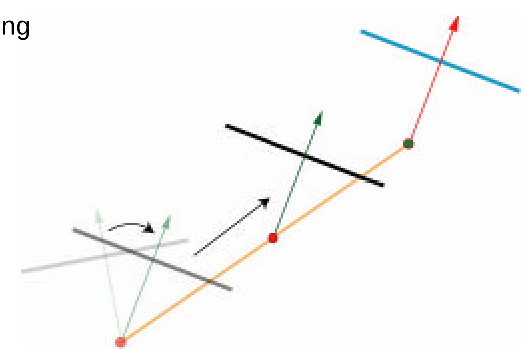
Visual Servoing (planar case)

Step1

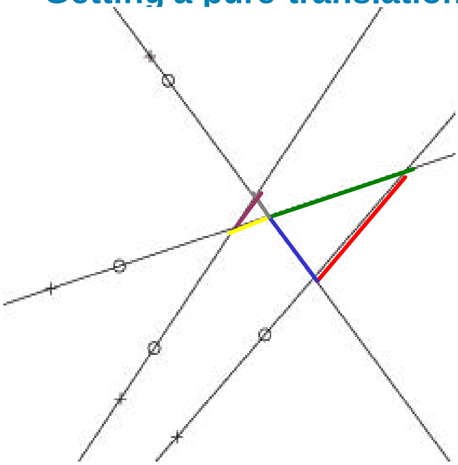
Find the right orientation exploiting the auto-epipole property

Step2

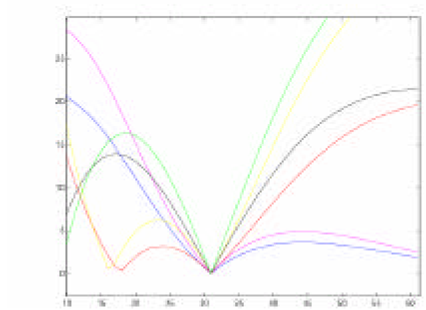
Follow the base-line until the target is reached



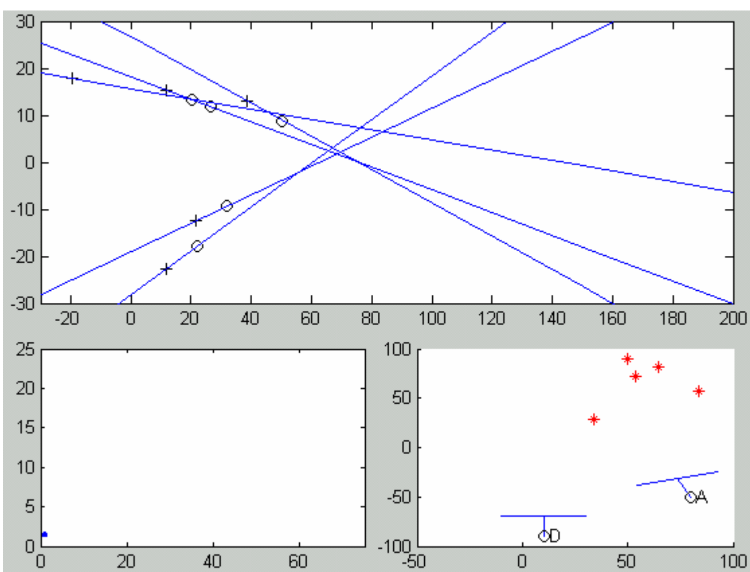
Getting a pure translation (step1)



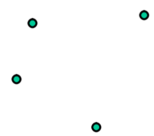
Control based on the the relative distances of intersecting points

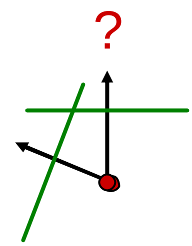


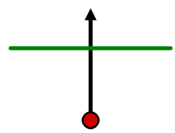
Control Variable: $\frac{d_{12}+d_{13}+d_{14}+\cdots}{N_d}$



Keeping features in the field of view!



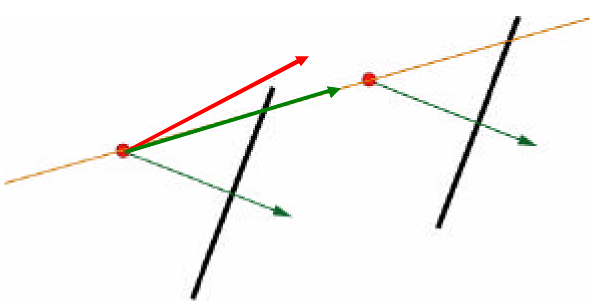




$$\begin{cases} \dot\theta = 0 \\ \dot z = -\mu(.) \\ \dot x = (-1)^j \mu(.) \end{cases}$$

2D Homing (step 2)

Follow the base line until the target is reached!



[P. Rives, INRIA]

Following the base line direction the epipole does not change

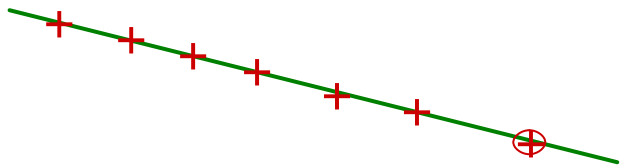
$$[Kt] = [e]$$

2D Homing

√ • Compensate the orientation

√ • Find the base-line direction

TO DO • Drive the camera to the target (one mode dof)



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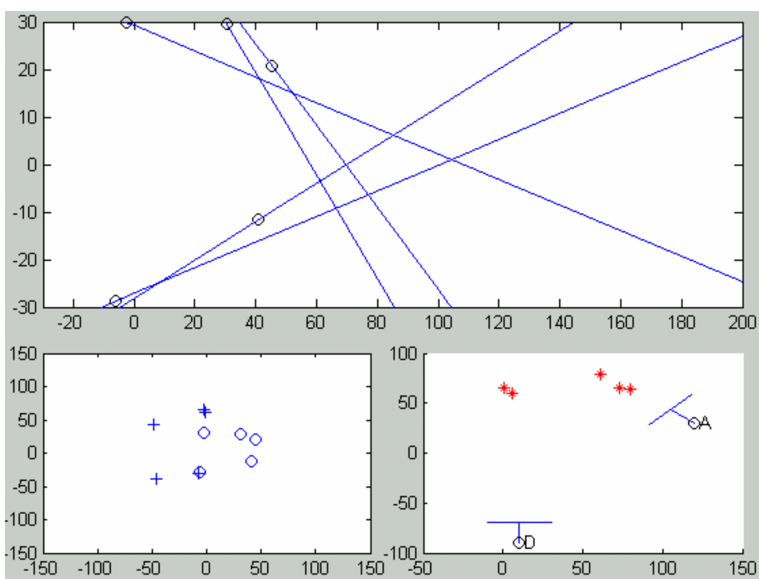
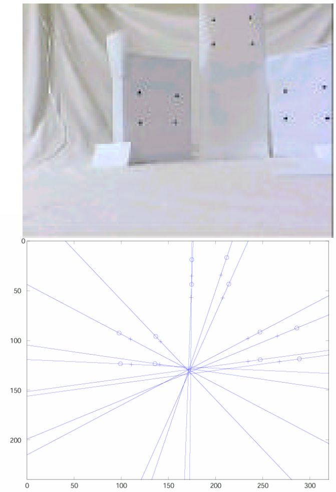
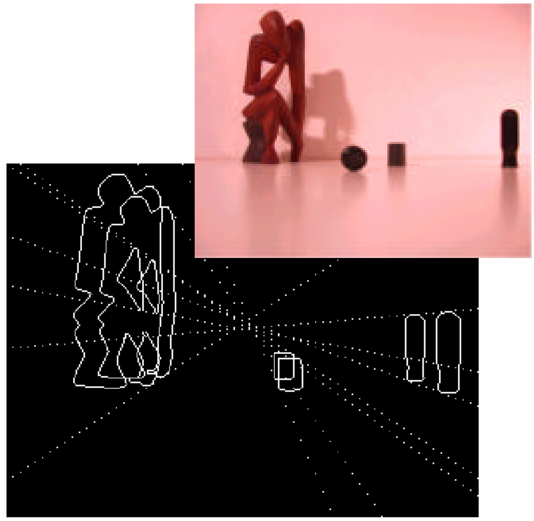


Controllo con Asservimento Visivo

The auto-epipole experiment



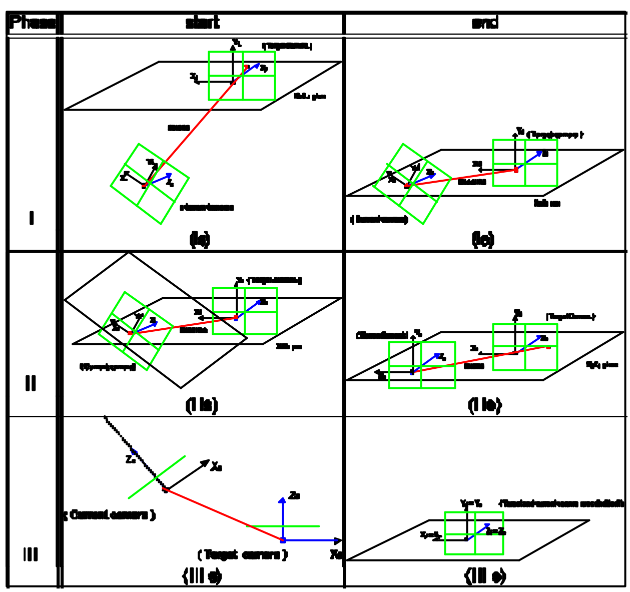


3-Phases Algorithm

Reaching the target plane

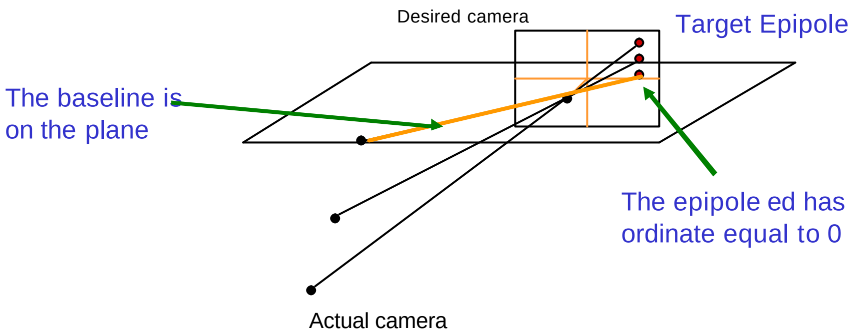
Getting main target coplanarity

the Planar Case



Reaching the target plane

Property 1: The epipole e_d has zero ordinate $e_{(d,v)} = 0$ *iff* the baseline belongs to the target plane. [Control Variable: e(d,v)]



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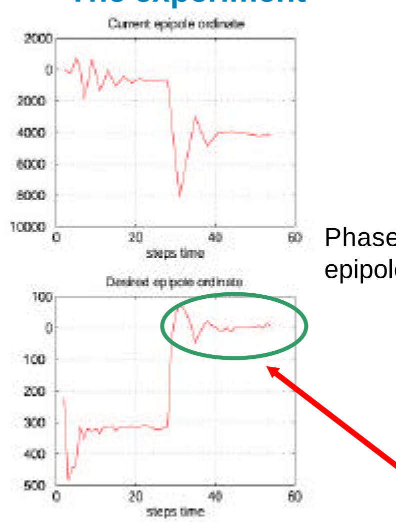
The experiment



Phase 1 epipole plots

Action Ib: *Steering O_c on the X_dZ_d plane.*

control error: the v–coordinate, in modulus, of the desired epipole

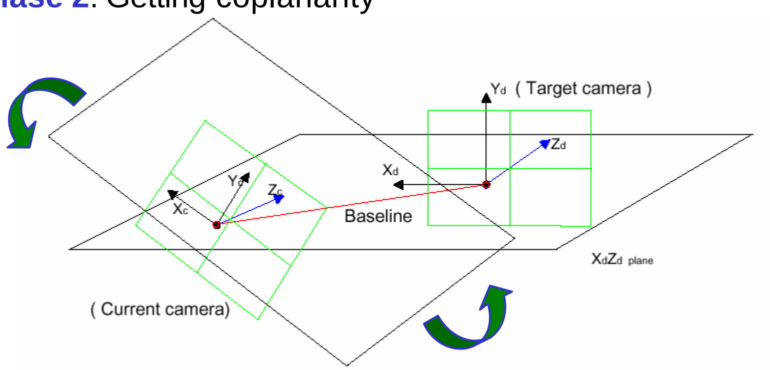
$$\alpha_i = \lambda |e_{(d,v)_i}|$$

$$H_i^{i+1}(\alpha_i) = Tr_y(\alpha_i)$$

being λ a proportional coefficient.

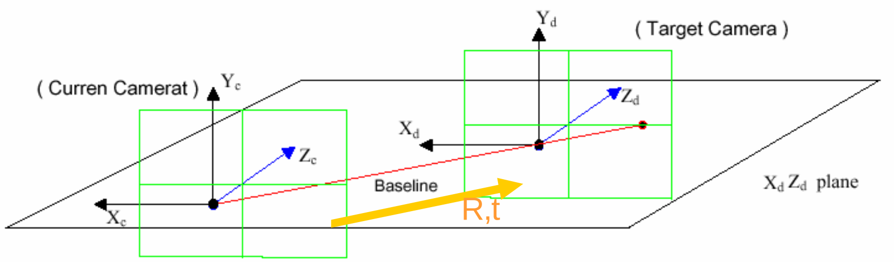
3–Phases Algorithm

Phase 2: Getting coplanarity



3–Phases Algorithm

Phase 3: 2D homing



Property 1 *The epipole* e_d *(*e_c*) has zero ordinate* $e_{(d,v)} = 0$ *(*$e_{(c,v)} = 0$*) if and only if the baseline belongs to the* $X_d Z_d$ *(*$X_c Z_c$*) plane.*

Property 2 *Refer to two cameras possibly with different internal camera matrices, the two epipoles* e_c *and* e_d *have zero* v*–component (*$e_{(c,v)} = 0, e_{(d,v)} = 0$*) if and only if the baseline belongs both to the* $X_c Z_c$ *and* $X_d Z_d$ *planes.*

Property 3 *Consider a fixed desired camera position. Then, when the current camera performs a complete rotation about the* Z_c *axis, the corresponding epipole* e_c *describes an ellipse centered in the principal point which reduces to a circle if* $\alpha_u = \alpha_v$*.*

Conclusions

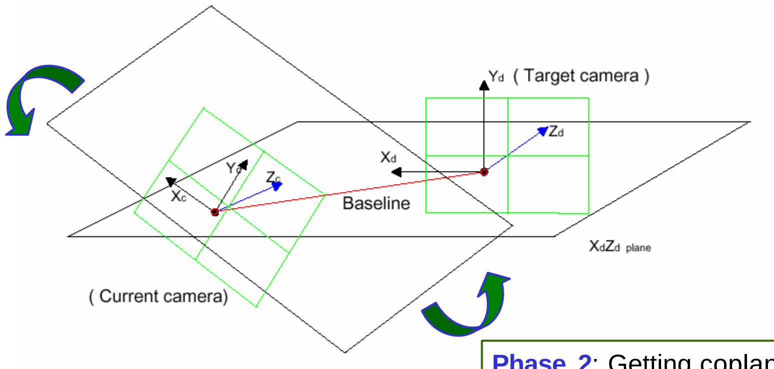
- The VS algorithm is based on the **epipole** kinematics
- Epipole estimation obtained from scene point features (or contours …)
- **Auto-epipole** approach (completely **uncalibrated** camera)

Next

- Epipole estimation must be robustly designed (image noise). **Trade-off** between accuracy and computational cost.
- Work is in progress for **contour based** visual servoing and comparison with existing methods
- Extension the auto-epipole to the fully actuated 3D camera motion
  - (sinergy with surface reconstruct using apparent contours and profiles [Cipolla and Giblin, 2000], [Chesi, Malis and Cipolla, 2000]
- Enforce the estimation of epipoles for the contour case.
  - Multipleview geometry [Hartley and Zisserman, 2000]
- Trajectory optimization (path planning) and **phase fusion**
- How to extend to **omnidirectional** cameras [Daniilidis] ?

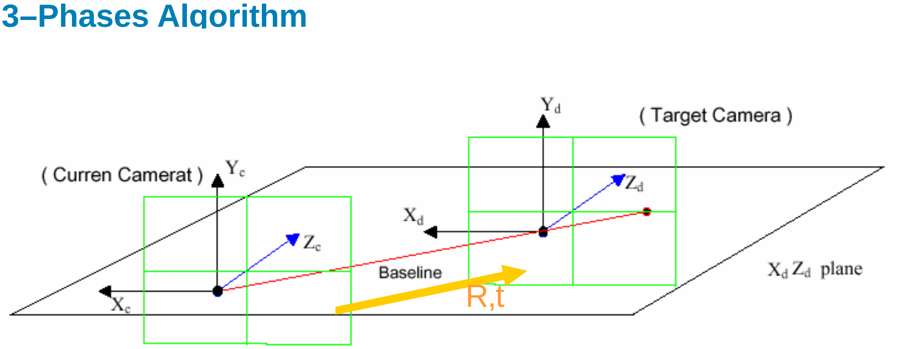
MORE SLIDES …..

3–Phases Algorithm



Phase 2: Getting coplanarity

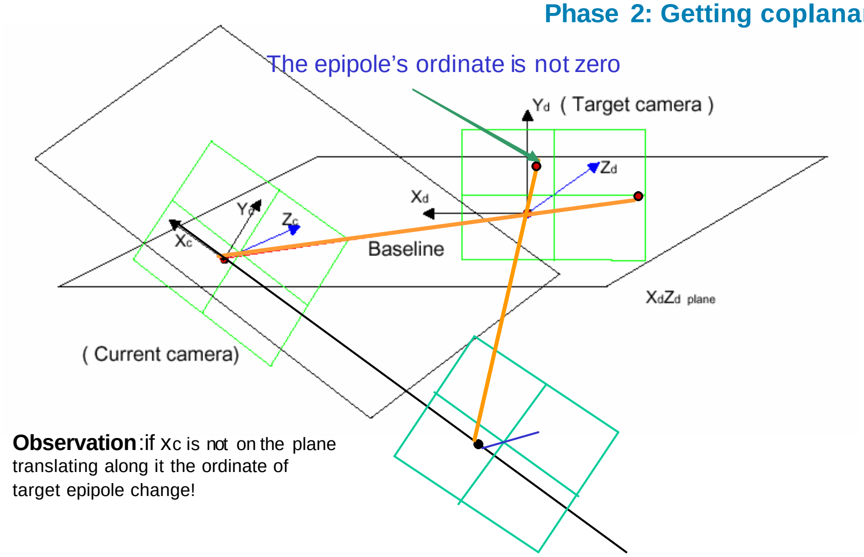
3–Phases Algorithm



Phase 3: 2D homing

Phase 2: Getting coplanarity

The epipole's ordinate is not zero



Observation: if Xc is not on the plane translating along it the ordinate of target epipole change!

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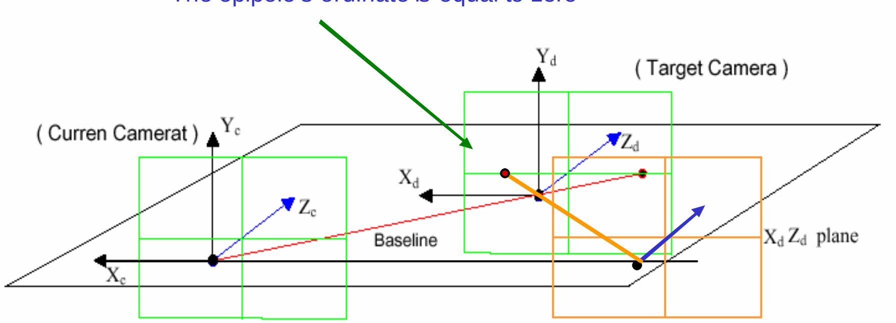
Phase 2: Getting coplanarity

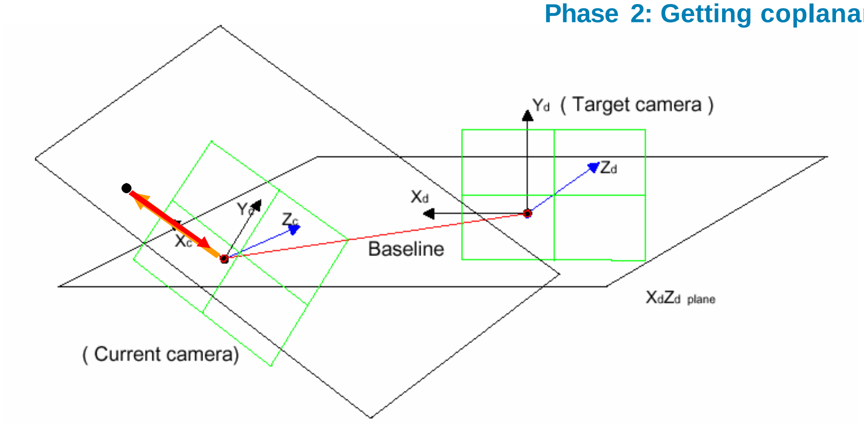
The epipole's ordinate is equal to zero



Observation: if Xc is on the plane translating along it the ordinate of target epipole remain to zero!

Controllo con Asservimento Visivo

Phase 2: Getting coplanarity



Idea:

Perform a test translation along xc and observe the target epipole

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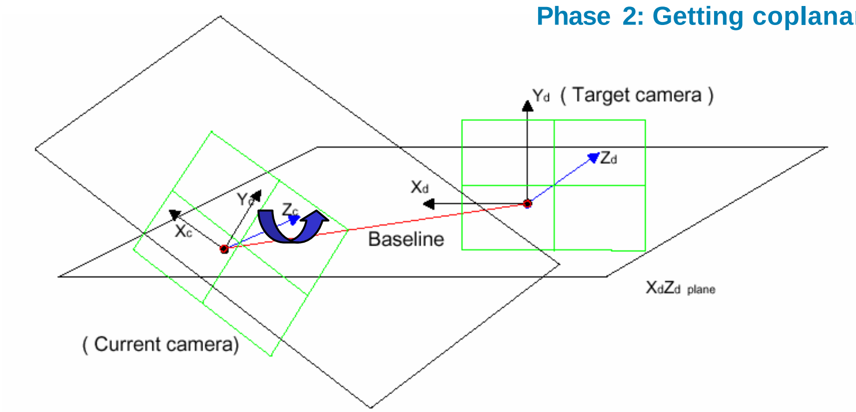
Phase 2: Getting coplanarity



Idea:

Rotate aroud Zc and repeat the process